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# INSIGHT INTO RADIOLOGICAL IMAGES

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## Abstract

Radiographs are two-dimensional projections of three-dimensional bodies. The resulting image has many drawbacks such as low contrast, blur, noise, and hidden parts. The objective of this paper is to discuss some of the peculiarities of radiological imaging in order to provide insight into the image formation process in radiology and its implications for modelling the system and processing radiological images. The following characteristics of the radiologic process are reviewed in this paper: functions and equations involved in image formation, factors that make the system non-linear and space-variant, noise sources, magnification phenomena, comparison of radiological processes, and psychophysics of vision relevant to image processing.

## INTRODUCTION

Radiographs are two-dimensional projections of three-dimensional bodies. This projection is obtained by passing a divergent X-ray beam, emitted by a finite-size source, through the body to be imaged and recording the attenuated rays on film. The resulting image has many drawbacks such as low contrast, blur, noise, and hidden parts. Much research has been done to analyze and improve the diagnostic quality of X-ray images (e.g. [1]-[3]).

The objective of this paper is to provide insight into the image formation process in radiology and its implications for modelling the system and processing radiological images. This insight was gained while investigating the design and realization of digital tomographic filters for radiographs to recover depth information [4]. While some of the results of this research have been published [5], [6], the background thinking should be emphasized more. Indeed, since radiological imaging has many interesting peculiarities, in this paper we have collected and explained some of them and discussed their implications. We trust that this paper will lead to a better understanding of the system and in the future will give rise to new solutions or processes. These will become extremely important when medical image communication systems are implemented in hospitals [7].

## IMAGE FORMATION IN RADIOLOGY

The radiologic process consists of a sequence of transformations intimately related in that the result of one forms the input to the next [1]. The transformations and degradations

introduced at each stage have been studied in the literature in great detail from the view-point of diagnostic image quality. Figure 1 shows a block diagram of the radiologic process. In general, block inputs and outputs are two-dimensional functions representing distributions of intensities. The operation or transformation in each block can be represented by mathematical equations relating the output to the input. There are many factors, especially those which are random in nature, which are considered to be noise and they are modelled by a perturbing noise source at the output of each block. The manner in which the noise is combined with the image depends on how the image has been formed and the nature of the noise process. Usually additive or multiplicative noise is assumed (e.g. [3]).

The first two blocks represent the generation of X-rays. The electron gun consists of a cathode emitting electrons which are focused and accelerated at high speed towards the anode. The region in the target where the X-rays and heat are produced is called the focal spot. The angle formed by the target surface and the direction of the centre X-ray is referred to as the target angle. Many studies have been published about the characteristics of focal spots in X-ray tubes (e.g. [9]). The shape and size of focal spots have been determined as well as their modulation transfer functions (MTF) and impulse responses or point spread functions (PTF), both theoretically and experimentally. The MTF of a focal spot resembles a Gaussian function. It is usually double-peaked and this introduces phase shifts in the image [10].

The interaction of X-rays with matter may be modelled by

$$I(z) = I(0) \cdot e^{-\int_0^z \mu(z) dz} \quad (1)$$

where  $I(z)$  is the intensity of a narrow X-ray beam from a point source as a function of the distance  $z$  in the direction of propagation and  $\mu(z)$  is a total attenuation coefficient in that direction.

X-rays propagate in straight lines. This fact controls the size, shape, and position on the radiographic film of the shadow or image of the various structures of the object being exposed. Due to the diverging nature of the X-rays emitted by the focal spot, the image of the object is magnified. With reference to Figure 2, the magnification for the layer at depth  $z_1$  is

$$m_1 = \frac{d_1 + d_2}{d_1} \quad (2)$$

which is a constant for each layer parallel to the film plane.

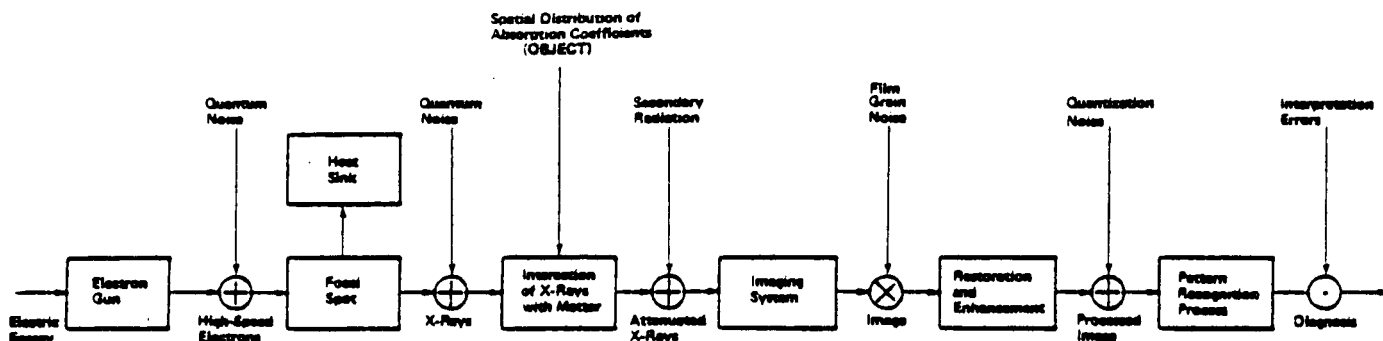


Figure 1. Block Diagram for the Radiologic Process

The X-ray image intensity distribution that reaches the imaging system is a function of both the object and the X-ray intensity distribution in the focal spot. Let  $I_0(x_0, y_0; x, y)$  be the X-ray intensity emitted from the point  $(x_0, y_0)$  in the focal spot towards the point  $(x, y)$  in the film plane. The spatial distribution of absorption coefficients in the object is denoted by  $\mu_L(z)$ , which is defined along a line  $L$  from  $(x_0, y_0)$  to  $(x, y)$ . The interaction between these two functions or inputs to the system can then be modelled by the following integral equation:

$$I(x, y) = \iint_{F.S.} I_0(x_0, y_0; x, y) e^{-\int_L \mu_L(z) dz} dx_0 dy_0 \quad (3)$$

which is a generalization of (1) and is obtained by integrating over the region of the focal spot (denoted here by F.S.). This equation is valid with any exposure function  $I_0$  and any three-dimensional object, in general.

From (3) it is clear that the radiologic process is linear with respect to  $I_0$  and non-linear with respect to the attenuation coefficients  $\mu$ . Nevertheless, an approximation can be made because the values of the linear attenuation coefficients, or at least their variations from point to point, are small and the exponential in (3) can be approximated by the linear terms of its Taylor series expansion [11]. Once the system is linearized it can be described by convolution integrals if the system is also space-invariant. However, the radiologic system is space-variant for several reasons, such as the divergent nature of the X-ray beam, the superposition of images of the layers in the object, the lack of parallelism of the focal spot and film planes and the change of the X-ray intensity emitted from the focal spot with direction.

Some solutions can be devised to make the space-variant problem tractable. The effect of the divergent nature of the X-ray beam when it reaches the film is that the intensity has been distorted according to the inverse square law. Since the consequences of this effect are deterministic, the intensity in the image can be corrected with image processing algorithms. Nevertheless, if the distances to be considered on the film plane are small, this effect can be neglected; because in radiology the focal spot to film distance is much greater than the focal spot size, say 1000:1. To deal with the problem of the varying intensity  $I_0(x_0, y_0; x, y)$  of the X-rays emitted from the focal spot if they are different in each direction, the image could be divided into small sections within which the impulse response could assumed to be constant. If a mathematical relationship between intensity and direction did exist, it would then be possible to correct the image intensity automatically, as in the case of the inverse-square law correction. However, in practical applications the intensity is normally the same in all directions, so that this correction is not necessary and the intensity becomes  $I_0(x_0, y_0)$ . The lack of parallelism of the focal spot and the film also makes the system space-variant. The PSF has different size and shape depending on the location of the pin-hole, even within the same layer. A solution has been proposed to correct for this problem [4], where a new (hypothetical) image is calculated by interpolation in a plane parallel to the focal spot, and thus, it has space-invariant properties. The equations of this transformation and the conditions under which it should be applied are given in [4].

$I_0(x_0, y_0)$  can be determined by exposing an object with a known distribution of absorption coefficients. If the object is a pin-hole, the

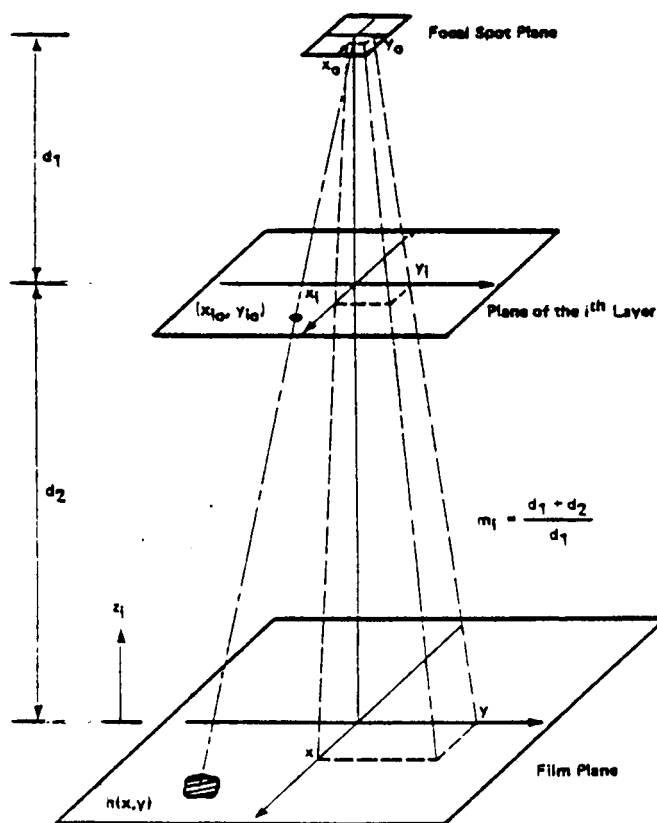


Figure 2. Coordinates in a Radiologic Process

system impulse response  $h(x, y)$  is obtained. It can be shown that the exposure function  $I_0(x_0, y_0)$  is then given by [4]:

$$I_0(x_0, y_0) = h\left(\frac{d}{d_1} x_0 - \frac{d_2}{d_1} x_0, \frac{d}{d_1} y_0 - \frac{d_2}{d_1} y_0\right)$$

where  $(x_0, y_0)$ ,  $d_1$  and  $d_2$  determine the position of the pin-hole, and  $d = d_1 + d_2$  (cf. Figure 2) [4]. Once the exposure function is known, the problem consists in recovering the spatial distribution of absorption coefficients based on a two dimensional projection, the image  $I(x, y)$ . This is not an easy problem. Conventional radiography masks the depth information by giving a shadow-cast image of the body, which contains hidden parts, blur due to the convolution with  $I_0$ , and noise.

#### COMPARISON OF RADIOLOGICAL PROCEDURES

Since conventional radiography does not provide information about the depths of details and structures, other radiological procedures have been invented over the years to overcome this problem [12]. Standard tomographic techniques produce images of certain parts of the body by blurring images from other parts. This is achieved by moving a point-like source and the recording film in a coupled manner, so that during the exposure only the parts of the body lying in one specific plane parallel to the film are always projected on the same place on the film, while the others are blurred. The layer whose image is in focus is referred to as the plane of cut or tomographic layer, and the overall resulting image is referred to as a tomogram.

Recently, another technique was proposed to produce a focusing effect similar to that of standard tomography but with no moving parts. Instead of moving the X-ray tube, the divergence of the X-ray beam and the finite size of the focal spot are used to advantage, and instead of moving the film, a filter is used to process a conventional radiograph. To understand this comparison better, consider a hypothetical point source moving all over the region on the actual source (the focal spot): by applying superposition the two sources are equivalent. The movement of this hypothetical point source would be analogous to the movement of an X-ray tube in standard tomography. However, in conventional radiography the film does not move, so the images of all the layers are blurred. Therefore, in order to convert a radiograph into a tomogram the radiographic image is processed by a filter that will produce an effect similar to that produced by the motion of the film in standard tomography. This is tomographic restoration (depth dependent): after processing the image of a particular layer is improved, while the others are degraded. This technique has been referred to as tomographic filtering or a tomographic filtration process (TFP) [4]-[6].

To explore how these various techniques compare to each other it is useful to consider the equation of radiology in the frequency domain where the space-domain convolutions become products. To make the transformation easier, in addition to the linearization previously discussed, it is convenient to make another approximation: if the 'size' of the exposure function  $I(\dots)$  is small compared to the distance from the source to the object, a differential length along the X-ray path ( $dz$ ) can then be replaced by the corresponding vertical differential length ( $dz_1$ ). Considering

these approximations (the complete derivation can be found in [4]), the final result is given by:

$$G(f_x, f_y) = I_0 \delta(f_x, f_y) - \int_0^d H_1(f_x, f_y, z_1) F_{\mu}(f_x, f_y, z_1) dz_1 \quad (4)$$

where  $G(f_x, f_y)$  is the Fourier transform of the resulting image on the film,  $I_0$  is a constant,  $\delta(f_x, f_y)$  is the Dirac delta function,  $H_1(f_x, f_y, z_1)$  is the transfer function of the  $i$ th layer, at a distance  $z_1$  from the film, which depends on the type of radiological procedure being considered, and  $F_{\mu}(f_x, f_y, z_1)$  is the two-dimensional Fourier transform of the attenuation coefficients,  $\mu(x_1, y_1, z_1)$ , at depth  $z_1$ , as given in (5).

$$F_{\mu}(f_x, f_y, z_1) = \iint \mu\left(\frac{d-z_1}{d} x, \frac{d-z_1}{d} y, z_1\right) e^{-i2\pi(f_x x + f_y y)} dx dy \quad (5)$$

It can be shown that the transfer function of standard tomography is given by:

$$H_1(f_x, f_y, z_1) = \left[ \frac{z_1 - d}{z_1 - D_2} \frac{D_1}{d} \right]^2 \iint I_0 \left\{ \frac{z_1 - d}{z_1 - D_2} \frac{D_1}{d} x, \frac{z_1 - d}{z_1 - D_2} \frac{D_1}{d} y \right\} e^{-i2\pi(f_x x + f_y y)} dx dy$$

where  $D_1$  and  $D_2$  determine the position of the plane of cut [4]. Conventional radiology could be considered as a special case of standard tomography ( $D_2 = 0$ ):

$$H_1(f_x, f_y, z_1) = \left[ \frac{z_1 - d}{z_1} \right]^2 \iint I_0 \left\{ \frac{z_1 - d}{z_1} x, \frac{z_1 - d}{z_1} y \right\} e^{-i2\pi(f_x x + f_y y)} dx dy$$

Finally, the overall transfer function of a tomographic filtration process is given by:

$$H_1(f_x, f_y, z_1) = \frac{\left[ \frac{z_1 - d}{z_1} \right]^2 \iint I_0 \left\{ \frac{z_1 - d}{z_1} x, \frac{z_1 - d}{z_1} y \right\} e^{-i2\pi(f_x x + f_y y)} dx dy}{\left[ \frac{z_1 - d}{z_1} \right]^2 \iint I_0 \left\{ \frac{z_1 - d}{z_1} x, \frac{z_1 - d}{z_1} y \right\} e^{-i2\pi(f_x x + f_y y)} dx dy}$$

where  $z_1$  is the depth of the layer to be deblurred by the tomographic filter [4].

It is interesting to note that the mathematical equations of conventional radiology, standard tomography and tomographic filtering are similar. Nevertheless, there are fundamental physical differences among these methods [4], [5]. The function  $I_0(\dots)$  in conventional radiology and tomographic filtering is defined over an area called the focal spot and the edges of this intensity distribution are not sharp, as discussed previously. On the other hand, in tomography  $I(\dots)$  defines the movement of a point-like X-ray source which is turned on and off over a line which can be straight, circular, elliptical, spiral, hypocycloidal, etc. The nature of the processes themselves are also different. Indeed, the transfer functions of conventional radiology and standard tomography correspond to truly radiologic procedures, while the transfer function of a tomographic filtration process has a component (the denominator) which corresponds to an image processing operation (inverse filtering). This means that the errors and noise are of different nature in each case. In standard tomography additional blur or errors occur if the patient moves during the exposure or there are mechanical misadjustments. On the other hand, in a tomographic filtration process the effect of a patient moving is not so critical because the exposure time is much shorter, but the filtering process is not ideal in practice and noise is amplified by the inverse filter, especially at high frequencies where the gain is greater.

Since radiographs consist of the superposition of the images of many layers, to fully understand the effects of radiograph processing we must also consider the composition of the spectrum of the projected object. Since different layers suffer different magnifications during exposure, the corresponding two-dimensional Fourier transforms of their shadow images are scaled accordingly. Assuming that each layer has the same spectrum, the relative scalings during magnification are shown in Figure 3 (for simplicity they are shown in one dimension only). These different scalings of the shadow images of the layers in the object makes the processing of radiographs more interesting. For example, a low-pass filter would enhance the images of the layers closer to the focal spot, while a high-pass filter would enhance the images of layers closer to the film. With band-pass or spectral-shaping filters in general, selective enhancement of certain layers could be realized. This is referred to as tomographic enhancement.

Finally, another factor that must be considered in image processing when images are to be judged by the human eye is the psychophysics of vision [13]. For example, the human observer is more sensitive to some spatial frequencies than others and he is more sensitive to intensity errors in grey areas than in white areas. The mean-square-error criterion is in very poor accord with subjective evaluation. Phase accuracy is extremely important in image processing filters [14].

#### CONCLUSIONS

In this paper we have reviewed a number of peculiarities in the formation of radiological images. All the characteristics are related in that they jointly affect the properties of the radiological image. A thorough understanding of them will permit better models of the process and the design of novel and more valuable image processing techniques for radiographs.

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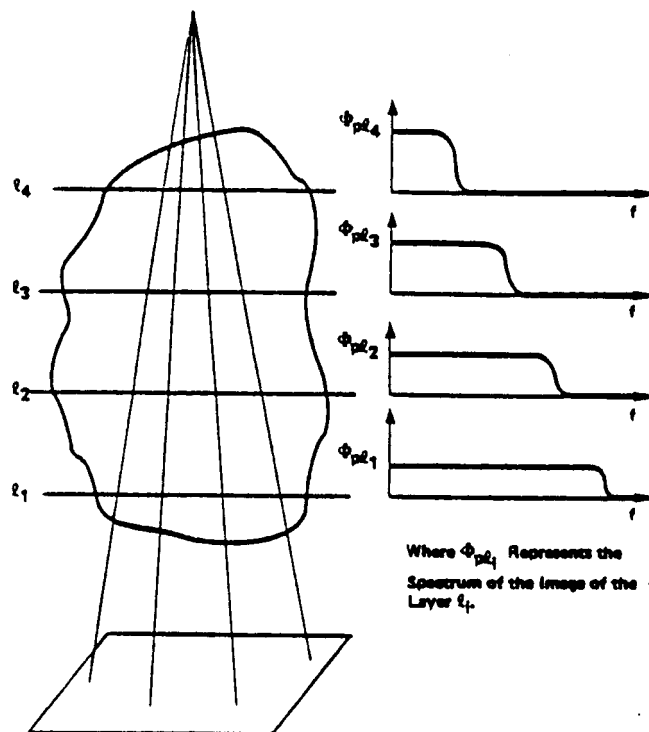


Figure 3. Scaling of the Spectra of the Images of Different Layers

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