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The Effects of Quantization Errors on Rotated Filters

STANLEY H. MNENEY, ANASTASIOS N. VENETSANOPOULOS, SENIOR MEMBER, IEEE, AND JOSÉ M. COSTA, MEMBER, IEEE

Abstract—**A digital filter that has been designed by rotation of the frequency response of a one-dimensional continuous filter, and then bilinearly transformed into a two-dimensional digital filter is called a rotated filter. Other useful filters such as circularly symmetric low-pass, high-pass, or bandpass filters can be obtained by parallel or cascade connection of rotated filters. These filters can be used in image processing and geophysics.**

Rotated filters are marginally stable if the rotation angle β satisfies $270° < \beta < 360°$, when designed from a stable one-dimensional continuous filter. A slight change in the coefficients has a significant effect on the stability of rotated filters. In this paper, the effects of quantization errors on the stability of rotated filters are investigated. A method to predict the stability state of rotated filters after coefficient quantization is given. A coefficient perturbation technique is discussed and used to stabilize a filter if found to be unstable.

For real coefficients, a region of guaranteed stability is defined for some forms of fixed-point and floating-point arithmetic. The effects of coefficient quantization and coefficient perturbation on the frequency response also are discussed.

I. Introduction

ROTATED FILTERS are designed by rotation of the transfer function of a stable one-dimensional continuous filter into a two-dimensional continuous filter, which is bilinearly transformed to form a two-dimensional digital filter [1]. These filters are marginally stable in the rotation angle β, where $270° < \beta < 360°$. Coefficient quantization affects both the frequency response and the stability of the filters. The effects of coefficient quantization on the frequency response of one-dimensional digital filters are discussed in [2] and [3]. The statistics of the errors at the output of two-dimensional digital filters are given in [4]. For rotated filters a slight change in the coefficients is expected to have a significant effect on their stability. In this paper, a method to predict the stability of rotated filters after coefficient quantization is presented. If unstable, the filters may be stabilized using the perturbation technique proposed in [1]. In fixed-point arithmetic, where sign-and-magnitude or one's complement representation is used, the region $270° < \beta_1 < \beta < \beta_2 < 360°$, is one in which the truncation errors will result in the stabilization of the filters. Here $\cos\beta_1 + \sin\beta_1 + p_i(T/2) = 0$, $-\cos\beta_2 - \sin\beta_2 + p_i(T/2) = 0$, and p_i is real. p_i is a pole in the one-dimensional continuous filter and T is the sampling interval. For floating-point arithmetic an identical region is defined, under the condition that the mantissa of the floating-point number has a two's complement representation.

The stability analysis for rotated filters is developed from the basic stability theorems quoted in Section II, which apply to two-dimensional digital filters in general. The design of rotated filters is summarized in Section III, and the regions of marginal stability are defined for an ideal filter. In Section IV, quantization error effects are investigated for fixed-point arithmetic, and in Section V, for floating-point arithmetic. The perturbation technique is discussed and applied to stabilize an unstable filter in Section VI. The effects of coefficient quantization and coefficient perturbation on the transfer function also are discussed.

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S. H. Mneney is with the Department of Electrical Engineering, University of Dar Es Salaam, Dar Es Salaam, Tanzania.

A. N. Venetsanopoulos is with the Department of Electrical Engineering, University of Toronto, Toronto, Canada.

J. M. Costa is with Bell-Northern Research, Ottawa, Ont., Canada.

II. Stability of Second-Order Two-Dimensional Digital Filters

The transfer function of a two-dimensional digital filter is given by

$$H(z_1, z_2) = \frac{A(z_1, z_2)}{B(z_1, z_2)} \triangleq \frac{\sum_{i=1}^{M_a} \sum_{j=1}^{N_a} a_{ij} z_1^{i-1} z_2^{j-1}}{\sum_{i=1}^{M_b} \sum_{j=1}^{N_b} b_{ij} z_1^{i-1} z_2^{j-1}} \tag{1}$$

where $b_{11} = 1$; z_1 and z_2 are unit delay operators; a_{ij} and b_{ij} are filter coefficients. $A(z_1, z_2)$ and $B(z_1, z_2)$ are mutually prime polynomials in z_1 and z_2. The filter is said to be *Bounded Input Bounded Output* (BIBO) stable if the impulse response is absolutely summable [5]. A number of stability theorems have been developed to express this stability condition in terms of the filter coefficients. Two of these theorems are quoted here as a necessary background in the stability analysis of rotated filters.

Theorem 1

A causal recursive filter with transfer function $H(z_1, z_2) = A(z_1, z_2)/B(z_1, z_2)$ where A and B are polynomials in z_1 and z_2, is stable iff there are no values of z_1 and z_2 such that $B(z_1, z_2) = 0$, $|z_1| \leq 1$, and $|z_2| \leq 1$. This theorem is given in [5] and a modification is discussed in [6].

Theorem 1 implies that the mapping of the unit disk

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$|z_1|\leq 1$ by the transformation $B(z_1, z_2)=0$ should not overlap with the unit disk $|z_2|\leq 1$ in the z_2 plane. This mapping requires considerable computation and simplifications are developed in [6] and [7].

Theorem 2

A causal recursive filter with transfer function $H(z_1, z_2) = A(z_1, z_2)/B(z_1, z_2)$ where A and B are polynomials in z_1 and z_2 is stable iff

a) The map of the unit circle of the z_1 plane $\{z_1; |z_1|=1\}$ into the z_2 plane according to the equation $B(z_1, z_2)=0$ lies outside the unit disk $\{z_2; |z_2|\leq 1\}$.

b) No point in the unit disk from the z_1 plane $\{z_1; |z_1| \leq 1\}$ maps into $z_2=0$ by the relationship $B(z_1, z_2)=0$.

Stability analysis for direct form higher order filters is unwieldy, since it involves a large number of coefficients. Design is usually limited to first- and second-order filters, which can be combined in parallel or in cascade. For a second-order filter the mapping $B(z_1, z_2)=0$, with complex coefficients in general, becomes a bilinear transformation; which maps circles into circles. Straight lines are circles with infinite radius [8]. In the case where the center of the mapped circle remains on the real axis, the points A_{z_2} and B_{z_2} mapped from $A_{z_1} \triangleq -1$ and $B_{z_1} \triangleq +1$, are sufficient to determine the filter stability. Here

$$A_{z_1} \triangleq -1 \quad \underset{\text{maps onto}}{\overset{B(z_1,z_2)=0}{\Longrightarrow}} \quad A_{z_2} = -\frac{b_{11}-b_{21}}{b_{12}-b_{22}} \tag{2a}$$

$$B_{z_1} \triangleq +1 \quad \underset{\text{maps onto}}{\overset{B(z_1,z_2)=0}{\Longrightarrow}} \quad B_{z_2} = -\frac{b_{11}+b_{21}}{b_{12}+b_{22}} \tag{2b}$$

and the center of the mapping is given in Appendix B of [1] as

$$C_{z_2} = \frac{b_{21}b_{22}^{*} - b_{11}b_{12}^{*}}{|b_{12}|^2 - |b_{22}|^2}. \tag{2c}$$

Hence, the following condition has to be satisfied:

$$\operatorname{Im}(b_{21}b_{22}^{*}) = \operatorname{Im}(b_{11}b_{12}^{*})$$

for the center to lie on the real axis.

III. Rotated Filters

Two techniques have been suggested for the design of rotated filters. The first approach was based on design of two-dimensional digital filters from one-dimensional continuous filters [1]. A perturbation technique was proposed as a method to stabilize the otherwise marginally stable filters.[1] A second method entailed the design of two-dimensional digital filters starting from one-dimensional discrete filters [9]. An optimization technique was adopted to produce filters with better stability properties. The coefficients were computed by means of nonlinear programming, subject to the stability constraints imposed on the coefficients. In general, rotated filters are marginally stable regardless of the design procedure. Only the first method is summarized for later application.

i) Design a stable one-dimensional continuous filter with a transfer function given by

$$H(s) = H_0 \frac{\prod_{i=1}^{m} (s+q_i)}{\prod_{i=1}^{n} (s+p_i)}. \tag{3}$$

If $\operatorname{Re}(p_i)>0$, then the pole $s=-p_i$ is in the left-hand plane of the s-plane and the filter is stable.

ii) Rotate $H(s)$ into a two-dimensional continuous filter by the transformation

$$s = -s_1' \sin\beta + s_2' \cos\beta.$$

This gives a rotated filter with a transfer function

$$H_1(s_1', s_2') = H_0 \frac{\prod_{i=1}^{m} [(s_2'\cos\beta - s_1'\sin\beta) + q_i]}{\prod_{i=1}^{n} [(s_2'\cos\beta - s_1'\sin\beta) + p_i]}. \tag{4}$$

iii) Apply a double bilinear transformation

$$s_1' = \frac{2}{T}\left(\frac{1-z_1}{1+z_1}\right) \text{ and } s_2' = \frac{2}{T}\left(\frac{1-z_2}{1+z_2}\right)$$

where T is the sampling interval, assumed to be the same in each direction. This gives a two-dimensional digital filter with a transfer function

$$H(z_1, z_2) = H_1 \prod_{i=1}^{M} \frac{(a_{11}^i + a_{21}^i z_1 + a_{12}^i z_2 + a_{22}^i z_1 z_2)}{(b_{11}^i + b_{21}^i z_1 + b_{12}^i z_2 + b_{22}^i z_1 z_2)} \tag{5}$$

where

$$\left.\begin{array}{l}
\left.\begin{array}{l}
a_{11}^i = \cos\beta - \sin\beta + q_i T/2 \\
a_{21}^i = \cos\beta + \sin\beta + q_i T/2 \\
a_{12}^i = -\cos\beta - \sin\beta + q_i T/2 \\
a_{22}^i = -\cos\beta + \sin\beta + q_i T/2
\end{array}\right\}, \quad 1 \leq i \leq m \\
a_{11}^i = a_{21}^i = a_{12}^i = a_{22}^i = 1, \qquad m < i \leq M \\
\left.\begin{array}{l}
b_{11}^i = \cos\beta - \sin\beta + p_i T/2 \\
b_{21}^i = \cos\beta + \sin\beta + p_i T/2 \\
b_{12}^i = -\cos\beta - \sin\beta + p_i T/2 \\
b_{22}^i = -\cos\beta + \sin\beta + p_i T/2
\end{array}\right\}, \quad 1 \leq i \leq n \\
b_{11}^i = b_{21}^i = b_{12}^i = b_{22}^i = 1, \qquad n < i \leq M.
\end{array}\right\} \tag{6}$$

$$H_1 \quad \text{constant}$$
$$M \triangleq \max(m, n).$$

iv) The required cutoff frequency is obtained by shifting the positions of p_i and q_i iteratively.

Stability of rotated filters can now be determined in terms of the cutoff frequency and the rotation angle. For the second-order rotated filter the points A_{z_2}, B_{z_2}, and C_{z_2} in (2) can be written in terms of the rotation angles using (6), refer to Fig. 1.

[1] Some authors use the term "marginally unstable." The two cases are equivalent and the impulse response will neither decay nor grow without bound. This condition is caused by the fact that rotated filters have a [illegible] the boundary of the unit disk, and no other singularities on the unit disk.

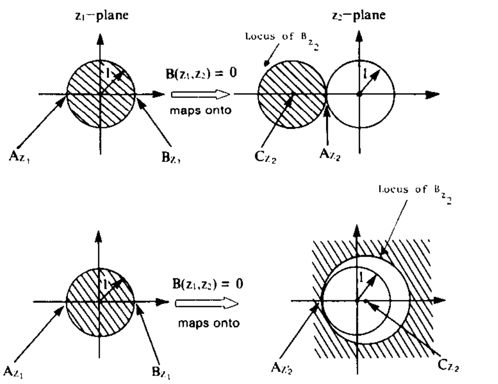

Fig. 1. Mapping that guarantees marginal stability.

$$A_{z_2} = -\left(\frac{-2\sin\beta}{-2\sin\beta}\right) = -1 \tag{7a}$$

$$B_{z_2} = -\left(\frac{\cos\beta + p_i T/2}{-\cos\beta + p_i T/2}\right) \tag{7b}$$

$$C_{z_2} = -\left(\frac{\mathrm{Re}(p_i T/2)}{\cos\beta + \mathrm{Re}(p_i T/2)}\right). \tag{7c}$$

The bilinear transformation $B(z_1, z_2) = 0$ maps circles onto circles. The point B_{z_2} lies anywhere on the circle with center C_{z_2}, since this is a mapping of $B_{z_1} = 1$, which lies on the unit circle in the z_1 plane. The point $A_{z_2} = -1$ is invariant irrespective of the rotation angle. The mapping of the unit circle from the z_1-plane by the bilinear transformation $B(z_1, z_2) = 0$ has a common point at $A_{z_2} = -1$, with the unit circle in the z_2-plane. This is the reason why at best rotated filters are marginally stable. C_{z_2} is always on the real axis. Theorem 2, as applied to rotated filters, can be simplified further. A filter is stable iff

i) A_{z_2}, C_{z_2} are real

ii) $|B_{z_2}| > |A_{z_2}| > 1$

iii) $|b_{11}/b_{21}| > 1$; no point inside the unit circle in the z_1-plane is mapped on the point $z_2 = 0$ in the z_2-plane.

For rotated filters condition i) is always true. In ii) equality holds for $|A_{z_2}| = 1$. This imposes a condition of marginal stability. For the other part of ii) an angle has to be determined for which

$$\left|\frac{\cos\beta + p_i T/2}{-\cos\beta + p_i T/2}\right| > 1 \tag{8a}$$

and from iii) an angle has to be determined for which

$$\left|\frac{\cos\beta - \sin\beta + p_i T/2}{\cos\beta + \sin\beta + p_i T/2}\right| > 1. \tag{8b}$$

Equation (8a) and (8b) are obtained by substituting (7a), (7b), and (6) in conditions ii) and iii).

Rotated filters are marginally stable if conditions (8a) and (8b) are satisfied. (8a) and (8b) are satisfied for $\mathrm{Re}(p_i T/2) > 0$ and $270° < \beta < 360°$. This condition implies that when the design of the rotated filter starts from a stable one-dimensional continuous filter, the rotation angle should be between 270° and 360° for marginal stability. It can also be noted that (8a) and (8b) are satisfied for $\mathrm{Re}(p_i T/2) < 0$ and $90° < \beta < 180°$. If the design starts from a single pole filter, with a pole on the right-hand plane, a stable two-dimensional filter can be obtained for an angle which is between 90° and 180°.

IV. Effects of Quantization Errors on Stability

In implementation of digital filters the coefficients are stored with finite precision. There is a quantization error associated with each coefficient. Let α_{ij} and β_{ij} be two quantization errors of the coefficients of (1) and $\bar{a}_{ij}$ and $\bar{b}_{ij}$ be the quantized coefficients such that

$$\bar{a}_{ij} = a_{ij} + \alpha_{ij}$$
$$\bar{b}_{ij} = b_{ij} + \beta_{ij}, \qquad i = 1,2, \quad j = 1,2 \text{ for a second-order filter} \tag{10}$$

for fixed-point arithmetic. The constants $\bar{a}_{ij}$, $\bar{b}_{ij}$, a_{ij}, b_{ij}, α_{ij}, and β_{ij} are complex in general. Quantization of the coefficients changes the positions of the poles and zeros from the desired locations. For rotated filters a slight change in the coefficients is sufficient to change the stability status of the filter. For second-order filters the transformation $\bar{B}(z_1, z_2) = 0$ is still bilinear.[2] The points $B_{z_1} = +1$ and $A_{z_1} = -1$ in the z_i-plane will be mapped to new positions such that

$$\bar{A}_{z_1} = -1 \ \underset{\text{maps onto}}{\overset{\bar{B}_1(z_1, z_2)=0}{\Longrightarrow}} \ \bar{A}_{z_2} = -\left(\frac{\bar{b}_{11} - \bar{b}_{21}}{\bar{b}_{12} - \bar{b}_{22}}\right) = -\left(\frac{2\sin\beta + \beta_{11} - \beta_{21}}{-2\sin\beta + \beta_{12} - \beta_{22}}\right) \tag{11}$$

$$\bar{B}_{z_1} = +1 \ \underset{\text{maps onto}}{\overset{\bar{B}(z_1, z_2)=0}{\Longrightarrow}} \ \bar{B}_{z_2} = -\left(\frac{\bar{b}_{11} + \bar{b}_{21}}{\bar{b}_{12} + \bar{b}_{22}}\right) = -\left(\frac{\cos\beta + p_i T/2 + \frac{1}{2}(\beta_{11} + \beta_{21})}{-\cos\beta + p_i T/2 + \frac{1}{2}(\beta_{12} + \beta_{22})}\right). \tag{12}$$

For a complex coefficient the real and imaginary parts are each truncated separately. For rounding $\mathrm{Im}(\beta_{ij})$ and $\mathrm{Re}(\beta_{ij})$ vary from $-q/2$ to $q/2$, while their magnitudes vary from 0 to q, for truncation. The quantity q is the quantization interval and is equal to 2^{-l}, where l is the selected register length. The point $\bar{A}_{z_2}$ is the most critical point in the stability analysis after quantization of the coefficients. $\bar{A}_{z_2}$ can be written as

$$\bar{A}_{z_2} = -\left(1 + \frac{\beta_{11} - \beta_{21} - \beta_{12} + \beta_{22}}{-2\sin\beta + \beta_{12} - \beta_{22}}\right). \tag{13}$$

Since $b_{11} = 1.0$ due to normalization of coefficients $\beta_{11} = 0$.

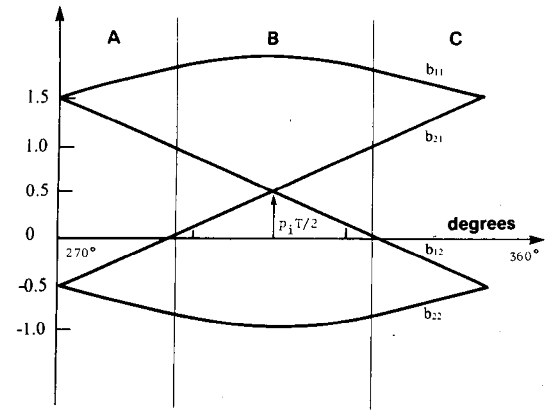

Fig. 2. Coefficients of rotated filters with p_i real and $270° < \beta < 360°$.

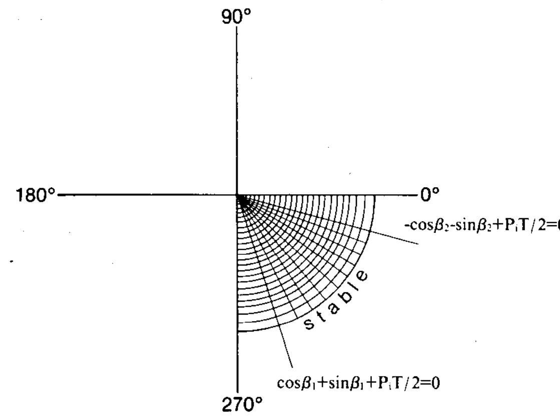

Fig. 3. Stability region of rotated filters with real coefficients.

Let

$$\epsilon \triangleq \frac{-\beta_{21}-\beta_{12}+\beta_{22}}{-2\sin\beta+\beta_{12}-\beta_{22}} \tag{14}$$

then

$$\bar{A}_{z_2} = -(1+\epsilon).$$

The stability parameter ϵ is a complex random variable. When $\mathrm{Re}[\epsilon]<0$, the transformation $B(z_1, z_2)=0$ does not cause an overlap of the mapping of the z_1-plane unit circle into the z_2-plane unit circle. The resulting filter is, therefore, stable. When $\mathrm{Re}[\epsilon]>0$ the filter is unstable. The parameter ϵ can be used to determine the stability status of the filter.

It is known [2] that for sign and magnitude representation and one's complement representation in fixed-point arithmetic, the truncation error is negative for positive numbers and positive for negative numbers. For real p_i the coefficients of a rotated filter have the characteristics shown in Fig. 2 in the region of marginal stability. The characteristics are summarized below for real $p_iT/2>0$ and $270° < \beta < 360°$.

TABLE I
THE COEFFICIENTS OF ROTATED FILTERS

θ	285°	315°	345°[a]	195°	225°	255°
a_{11}	1.2427902	1.2173309	1.2427893	-0.2490057	2.0000029	1.3570623
a_{12}	0.9227570	0.4346644	0.0484048	-6.3933916	5.6012859	1.8277283
a_{21}	0.0484031	0.4346620	0.9227546	1.3973694	-1.6012707	-0.3994782
a_{22}	-0.2716300	-0.3480055	-0.2716304	-4.7470169	2.0000124	0.0711879
b_{11}	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
b_{12}	0.6799673	0.2173329	-0.1943851	-5.1443853	4.6012812	1.4706650
b_{21}	-0.1943869	0.2173305	0.6799651	2.6463747	-2.6012745	-0.7565408
b_{22}	-0.5144200	-0.5653370	-0.5144203	-3.4980106	1.0000086	-0.2858747

[a] These filters are unstable.

[2], a region of guaranteed stability does not exist. A similar problem arises when rounding error is considered. Given the register length, the quantization errors on the

Region A

$b_{11}=1.0$ $\quad \beta_{11}=0$
$b_{21}<0$ $\quad \beta_{21}>0$
$b_{12}>0$ $\quad \beta_{12}<0$
$b_{22}<0$ $\quad \beta_{22}>0$

$$\epsilon_A = \frac{-|\beta_{21}|+|\beta_{12}|+|\beta_{22}|}{-2\sin\beta} \qquad \sin\beta\neq 0.$$

Region B

$b_{11}=1.0$ $\quad \beta_{11}=0$
$b_{21}>0$ $\quad \beta_{21}<0$
$b_{12}>0$ $\quad \beta_{12}<0$
$b_{22}<0$ $\quad \beta_{22}>0$

$$\epsilon_B = \frac{|\beta_{21}|+|\beta_{12}|+|\beta_{22}|}{-2\sin\beta} \qquad \sin\beta\neq 0.$$

Region C

$b_{11}=1.0$ $\quad \beta_{11}=0$
$b_{21}>0$ $\quad \beta_{21}<0$
$b_{12}<0$ $\quad \beta_{12}>0$
$b_{22}<0$ $\quad \beta_{22}>0$

$$\epsilon_C = \frac{|\beta_{21}|-|\beta_{12}|+|\beta_{22}|}{-2\sin\beta} \qquad \sin\beta\neq 0.$$

Here $\sin\beta$ is much greater than $\beta_{12}-\beta_{22}$, since β_{12} and β_{22} are quantization errors which have small values in practice. It is seen that $\epsilon_B>0$ and hence the filter is stable for $\beta_1<\beta<\beta_2$, where $\cos\beta_1+\sin\beta_1+p_iT/2=0$ and $-\cos\beta_2-\sin\beta_2+p_iT/2=0$, as shown in Fig. 3.

For real $p_i(T/2)<0$ the region of marginal stability is $90°<\beta<180°$. It can similarly be shown that in this region the stability parameters ϵ_A, ϵ_B, and ϵ_C are not guaranteed to be positive.

For two's complement representation, where the errors for both positive and negative numbers are always negative coefficients can be computed exactly. It is, therefore, possible to test the sign of the stability parameter on each design, and especially in the range where the filter stability is not guaranteed.

Table I contains a set of coefficients for filters designed from a stable one-dimensional continuous filter, as in Section III, for certain rotation angles β. Tables II and III give the truncation errors for various register lengths and compute the stability parameter ϵ for sign-and-magnitude representation of fixed-point arithmetic. The region of guaranteed stability is $289°<\beta<341°$. It is clear from

TABLE II
COEFFICIENT QUANTIZATION ERRORS AT $\beta=285°$

Register Length	4	6	8	12	16
a_{11}	-0.2427902	-0.1177902	-0.0240402	-0.0006027	-0.0001144
a_{12}	-0.4227570	-0.0477570	-0.0165070	-0.0008820	-0.0000275
a_{21}	-0.0484031	-0.0484031	-0.0171531	-0.0015281	-0.0000633
a_{22}	0.2716300	0.0216300	0.0216300	0.0001456	0.0000235
b_{11}	0.0	0.0	0.0	0.0	0.0
b_{12}	-0.6799673	-0.0549673	-0.0237173	-0.0002798	-0.0000356
b_{21}	0.1943869	0.0693869	0.0068869	0.0010275	0.0000510
b_{22}	0.5144200	0.0144200	0.0144200	0.0007482	0.0000157
ϵ	$2.3\text{x}10^{-7}$	$2.15\text{x}10^{-7}$	$1.62\text{x}10^{-7}$	$2.58819\text{x}10^{-7}$	$1.553\text{x}10^{-7}$

TABLE III
COEFFICIENT QUANTIZATION ERRORS AT $\beta=315°$

Register Length	4	6	8	12	16
a_{11}	-0.2173309	-0.0923309	-0.0298309	-0.0005341	0.0000458
a_{12}	-0.4346644	-0.0596644	-0.0284144	-0.0010707	0.0000941
a_{21}	-0.4346620	-0.0596620	-0.0284120	-0.0010682	0.0000917
a_{22}	0.3480055	0.0980055	0.0041555	0.0003492	0.0001051
b_{11}	0.0	0.0	0.0	0.0	0.0
b_{12}	-0.2173329	-0.2173329	-0.0298329	-0.0005360	-0.0000477
b_{21}	-0.2173305	-0.2173305	-0.0298305	-0.0005336	-0.0000453
b_{22}	0.5653370	0.0653370	0.0028370	0.0008839	0.0000294
ϵ	1.58	0.4418	$4.410\text{x}10^{-2}$	$1.383\text{x}10^{-3}$	$8.655\text{x}10^{-5}$

Table III that filters designed with the rotation angle 315° and $p_iT/2=0.62$ fall in the region of guaranteed stability, since $\text{Re}[\epsilon]>0$. For $\beta=285°$, $\text{Re}[\epsilon]>0$, but smaller than the least significant digit of the coefficients. In this case ϵ cannot be taken as a reliable stability sensor, since the arithmetic computation of ϵ does not guarantee an accuracy greater than that in which the coefficients are given. In such cases the filter coefficients should be perturbed to guarantee stability. This technique is discussed in Section VI.

V. FILTERS IMPLEMENTED USING FLOATING-POINT ARITHMETIC

Floating-point numbers are of the form $2^a \cdot b$ [6]. The exponent a is an integer in binary form. The variable b is the mantissa and is between 1/2 and 1. The variables a and b are represented with limited wordlength, introducing quantization errors. A real number v is approximated by the number v_t in floating-point arithmetic. Thus $v_t=v(1+k)$, where k is the truncation or rounding error. Both addition and multiplication in floating-point arithmetic introduce errors. The sum $(v_1+v_2)_t=(v_1+v_2)(1+\rho)$ and the product $(v_1 \cdot v_2)_t=(v_1 \cdot v_2)(1+\delta)$, where ρ and δ are quantization errors.

The second-order two-dimensional digital filter with infinite precision arithmetic has a recursive equation given by

$$y_{nm}=\sum_{i=1}^{2}\sum_{j=1}^{2} a_{ij}x_{n-i+1,m-j+1}-\sum_{\substack{i=1\\i+j\neq 2}}^{2}\sum_{j=1}^{2} b_{ij}y_{n-i+1,m-j+1}. \tag{15}$$

Let α_{ij} and β_{ij} be the errors introduced in approximating the coefficients a_{ij} and b_{ij} by the floating-point numbers. If $\text{fl}(\cdot)$ denotes floating-point arithmetic the recursion equation can be written as

$$\bar{y}_{nm}=\text{fl}\left[\sum_{i=1}^{2}\sum_{j=1}^{2} a_{ij}(1+\alpha_{ij})x_{n-i+1,m-j+1}\right.$$
$$\left.-\sum_{\substack{i=1\\i+j\neq 2}}^{2}\sum_{j=1}^{2} b_{ij}(1+\beta_{ij})\bar{y}_{n-i+1,m-j+1}\right]. \tag{16}$$

The order of computation of (16) is such that the products $a_{ij}(1+\beta_{ij})x_{n-i+1,m-j+1}$ and $b_{ij}(1+\beta_{ij})\bar{y}_{n-i+1,m-j+1}$ are performed first. The sums are then performed and finally the difference to give $\bar{y}_{nm}$. The computation order and the errors introduced are summarized in the flow chart of Fig. 4: where

α_{ij} and β_{ij} $i,j=1,2$ are coefficient quantization errors;
δ_{ij} and λ_{ij} $i,j=1,2$ are errors due to multiplication;
ξ_i and ϵ_i $i=1,2$ are errors due to addition.

Let

$$\phi_{11}=(1+\alpha_{11})(1+\delta_{n,m})(1+\xi_1)(1+\xi_2)(1+\xi_3)(1+k)$$
$$\phi_{21}=(1+\alpha_{21})(1+\delta_{n-1,m})(1+\xi_1)(1+\xi_2)(1+\xi_3)(1+k)$$
$$\phi_{12}=(1+\alpha_{12})(1+\delta_{n,m-1})(1+\xi_2)(1+\xi_3)(1+k)$$
$$\phi_{22}=(1+\alpha_{22})(1+\delta_{n-1,m-1})(1+\xi_3)(1+k)$$

and

$$\theta_{11}=1$$
$$\theta_{21}=(1+\beta_{21})(1+\lambda_{n-1,m})(1+\epsilon_1)(1+\epsilon_2)(1+k)$$
$$\theta_{12}=(1+\beta_{12})(1+\lambda_{n,m-1})(1+\epsilon_1)(1+\epsilon_2)(1+k)$$
$$\theta_{22}=(1+\beta_{22})(1+\lambda_{n-1,m-1})(1+\epsilon_2)(1+k).$$

The random variables α, δ, ξ, k, β, λ, and ϵ are quantization errors, which vary from $(-2^{-l}/2)$ to $(2^{-l}/2)$ for rounding, while their magnitudes vary from 0 to 2^{-l} for truncation. The register length is given by l. The products and squares of these random variables are negligible compared to the values themselves. The following approximations are

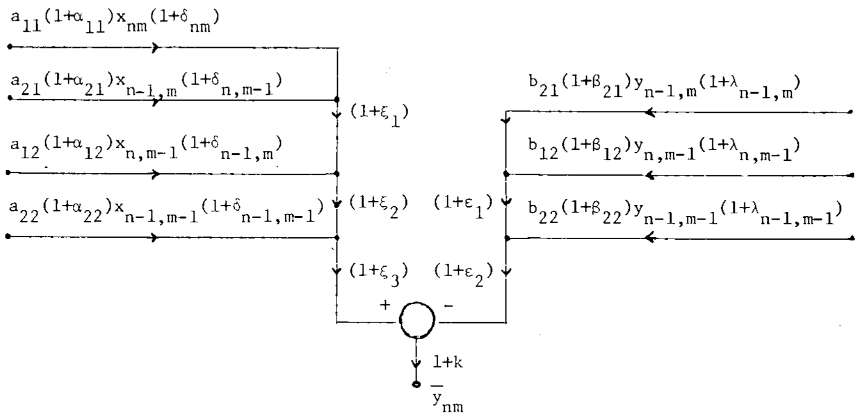

Fig. 4. Computation order for a recursive filter using floating-point arithmetic.

thus useful:

$$\phi_{11} \simeq 1+K_{11} = 1+\alpha_{11}+\delta_{n,m}+\xi_1+\xi_2+\xi_3+k$$
$$\phi_{21} \simeq 1+K_{21} = 1+\alpha_{21}+\delta_{n-1,m}+\xi_1+\xi_2+\xi_3+k$$
$$\phi_{12} \simeq 1+K_{12} = 1+\alpha_{12}+\delta_{n,m-1}+\xi_2+\xi_3+k$$
$$\phi_{22} \simeq 1+K_{22} = 1+\alpha_{22}+\delta_{n-1,m-1}+\xi_2+k$$

and

$$\theta_{11} = 1.0+L_{11} \quad L_{11}=0.0$$
$$\theta_{21} \simeq 1+L_{21} = 1+\beta_{21}+\lambda_{n-1,m}+\epsilon_1+\epsilon_2+k$$
$$\theta_{12} \simeq 1+L_{12} = 1+\beta_{12}+\lambda_{n,m-1}+\epsilon_1+\epsilon_2+k$$
$$\theta_{22} \simeq 1+L_{22} = 1+\beta_{22}+\lambda_{n-1,m-1}+\epsilon_2+k.$$

Thus (16) can be written as

$$\bar{y}_{nm} = \sum_{i=1}^{2}\sum_{j=1}^{2} a_{ij}(1+K_{ij})x_{n-i+1,m-j+1} - \sum_{\substack{i=1\\i+j\neq 2}}^{2}\sum_{j=1}^{2} b_{ij}(1+L_{ij})y_{n-i+1,m-j+1}. \quad (17)$$

The z-transform of the transfer function is given by

$$\bar{H}(z_1,z_2) = \frac{\sum_{i=1}^{2}\sum_{j=1}^{2} a_{ij}(1+K_{ij})z_1^{i-1}z_2^{j-1}}{\sum_{i=1}^{2}\sum_{j=1}^{2} b_{ij}(1+L_{ij})z_1^{i-1}z_2^{j-1}}. \quad (18)$$

Let $\bar{a}_{ij} = a_{ij}(1+K_{ij})$ and $\bar{b}_{ij} = b_{ij}(1+L_{ij})$, then

$$\bar{H}(z_1,z_2) = \frac{\sum_{i=1}^{2}\sum_{j=1}^{2} \bar{a}_{ij}z_1^{i-1}z_2^{j-1}}{\sum_{i=1}^{2}\sum_{j=1}^{2} \bar{b}_{ij}z_1^{i-1}z_2^{j-1}}. \quad (19)$$

Stability Theorems 1 and 2 still hold. The points A_{z_1} and B_{z_2} are mapped onto different points by a transformation $\bar{B}(z_1,z_2)=0$. The invariant point $A_{z_1}=A_{z_2}=-1$ is now given by

$$A_{z_2} = -\left(\frac{b_{11}(1+L_{11})-b_{21}(1+L_{21})}{b_{21}(1+L_{12})\;\; b_{22}(1+L_{22})}\right)$$

which after some algebra becomes

$$A_{z_2} = -\left[1+\frac{-L_{21}b_{21}-L_{12}b_{12}+L_{22}b_{22}}{-2\sin\beta+L_{12}b_{12}-L_{22}b_{22}}\right]. \quad (20)$$

A_{z_2} is of the form

$$A_{z_2} = -(1+\epsilon)$$

where

$$\epsilon = \frac{-L_{21}b_{21}-L_{12}b_{12}+L_{22}b_{22}}{-2\sin\beta+L_{12}b_{12}-L_{22}b_{22}} \quad (21)$$

and if $\mathrm{Re}(\epsilon)>0$ the filter is stable and if $\mathrm{Re}(\epsilon)<0$ the filter is unstable.

For floating-point arithmetic the mantissa can be represented using sign-and-magnitude, one's complement or two's complement representation [10]. Many machines use two's complement representation for the mantissa. In such a case [6] demonstrates that the errors are always negative. Using Fig. 2 for real $p_iT/2>0$

Region A

$$L_{21}<0 \qquad b_{21}<0 \qquad -L_{21}b_{21}<0$$
$$L_{12}<0 \qquad b_{12}>0 \qquad -L_{12}b_{12}>0$$
$$L_{22}<0 \qquad b_{22}<0 \qquad L_{22}b_{22}>0$$
$$\epsilon_A = \frac{-|L_{21}b_{21}|+|L_{12}b_{12}|+|L_{22}b_{22}|}{|2\sin\beta|} \qquad \sin\beta\neq 0.$$

Region B

$$L_{21}<0 \qquad b_{21}>0 \qquad -L_{21}b_{21}>0$$
$$L_{12}<0 \qquad b_{12}>0 \qquad -L_{12}b_{12}>0$$
$$L_{22}<0 \qquad b_{22}<0 \qquad L_{22}b_{22}>0$$
$$\epsilon_B = \frac{|L_{21}b_{21}|+|L_{12}b_{12}|+|L_{22}b_{22}|}{|2\sin\beta|}>0 \qquad \sin\beta\neq 0.$$

Region C

$$L_{21}<0 \qquad b_{21}>0 \qquad -L_{21}b_{21}>0$$
$$L_{12}<0 \qquad b_{12}<0 \qquad -L_{12}b_{12}<0$$
$$L_{22}<0 \qquad b_{22}<0 \qquad L_{22}b_{22}>0$$
$$\epsilon_C = \frac{|L_{21}b_{21}|-|L_{12}b_{12}|+|L_{22}b_{22}|}{|2\sin\beta|} \qquad \sin\beta\neq 0$$

In Region B, $\epsilon_B>0$ and the filter is guaranteed to be stable. In Regions A and C the filter may be unstable. Region B is bounded by $\beta_1<\beta<\beta_2$, where $\cos\beta_1+\sin\beta_1+p_iT/2=0$ and $-\cos\beta_2-\sin\beta_2+p_iT/2=0$. When $p_iT/2<0$ there is no region of guaranteed stability. The departure from the symmetry is due to the coefficient normalization factor b_{11}, which is now a negative quantity. This factor is positive for $p_iT/2>0$. Region B is a function of the pole position in the one-dimensional continuous filter and the desired cutoff frequency. It is easy to see that for $p_iT/2=0$, Region B is minimum. This region is maximum for $p_iT/2=1$.

VI. The Stabilization Technique

It is possible to stabilize rotated filters that are found to be unstable by perturbing the coefficients. Not all coefficients need be perturbed and the choice depends on the approach taken. The basis of this stabilization technique is given below.

A marginally stable filter is destabilized by the quantization error such that

$$\bar{A}_{z_2}=-\left(\frac{\bar{b}_{11}-\bar{b}_{21}}{\bar{b}_{12}-\bar{b}_{22}}\right)>-1. \qquad (23)$$

In this case the mapping of the unit circle from the z_1-plane onto the z_2-plane overlaps with the unit circle on the z_2-plane. In this case $\mathrm{Re}(\epsilon)<0$. $\mathrm{Re}(\epsilon)$ is given by (14). After quantization equality can be retained if $\mathrm{Re}(\epsilon)$ is subtracted from the right-hand side of (23):

$$\bar{A}_{z_2}=-\left(\frac{\bar{b}_{11}-\bar{b}_{21}}{\bar{b}_{12}-\bar{b}_{22}}\right)=-(1+\mathrm{Re}(\epsilon)). \qquad (24)$$

This "shrinking" of the unit circle retains the original condition of marginal stability as before quantization. The unit circle on the z_2-plane is hypothetically shrunk by $\mathrm{Re}(\epsilon)$, until it just touches the mapping of the unit circle from the z_1-plane onto the z_2-plane. For stability the two circles should not touch but have a gap of δ between them. Thus

$$\bar{A}_{z_2}=-\left(\frac{\bar{b}_{11}-\bar{b}_{21}}{\bar{b}_{12}-\bar{b}_{22}}\right)<-(1+\mathrm{Re}(\epsilon)+\delta). \qquad (25)$$

Let

$$\mu=\mathrm{Re}(\epsilon)+\delta$$

$$\bar{A}_{z_2}=-\left(\frac{\bar{b}_{11}-\bar{b}_{21}}{\bar{b}_{12}-\bar{b}_{22}}\right)<-(1+\mu). \qquad (26)$$

Different approaches of stabilizing a rotated filter are now given:

i) Divide the inequality (26) by $(1+\mu)$ to give

$$\tilde{A}_{z_2}=\frac{-(\bar{b}_{11}-\bar{b}_{21})}{\bar{b}_{12}(1+\mu)-\bar{b}_{22}(1+\mu)}<-1 \qquad (27)$$

by perturbing the coefficients $\bar{b}_{12}$ and $\bar{b}_{22}$ to $\tilde{b}_{12}=\bar{b}_{12}(1+\mu)$ and $\tilde{b}_{22}=\bar{b}_{22}(1+\mu)$, inequality (27) is guaranteed and the filter is stabilized.

ii) Add μ to both sides of inequality (26)

$$\tilde{A}_{z_2}=\bar{A}_{z_2}+\mu=\frac{-(\bar{b}_{11}-\bar{b}_{21})}{(\bar{b}_{12}-\bar{b}_{22})}+\mu<-1 \qquad (28)$$

by perturbing $\bar{b}_{11}$ and $\bar{b}_{21}$ to $\tilde{b}_{11}=\bar{b}_{11}-\mu\bar{b}_{12}$ and $\tilde{b}_{21}=\bar{b}_{21}-\mu\bar{b}_{22}$, inequality (28) is guaranteed and the filter is stabilized.

iii) By perturbing $\bar{b}_{11}$ and $\bar{b}_{21}$ to $\tilde{b}_{11}=\bar{b}_{11}-\mu\bar{b}_{12}+\mu\bar{b}_{22}$ and $\tilde{b}_{21}=\bar{b}_{21}$, inequality (28) is guaranteed and the filter is stabilized.

iv) By perturbing $\bar{b}_{11}$ and $\bar{b}_{21}$ such that $\tilde{b}_{11}=\bar{b}_{11}$ and $\tilde{b}_{21}=\bar{b}_{21}+\mu\bar{b}_{12}-\mu\bar{b}_{22}$, inequality (28) is guaranteed and the filter is stabilized.

The preferred approach is the one which produces the least distortion in the frequency response. The value of the derivative of the frequency response with respect to a coefficient is a measure of the sensitivity of the frequency response to changes in the values of the filter coefficients. The coefficients corresponding to the lowest sensitivity are perturbed. From (1) the coefficient sensitivities of the second-order filter with $M_a=N_a=M_b=N_b=2$ are

$$\frac{dH}{db_{11}}=M,\quad \frac{dH}{db_{21}}=Me^{-jw_1},\quad \frac{dH}{db_{12}}=Me^{-jw_2},$$

$$\frac{dH}{db_{22}}=Me^{-j(w_1+w_2)}$$

where

$$M=-\frac{|a_{11}+a_{21}e^{-jw_1}+a_{12}e^{-jw_2}+a_{22}e^{-j(w_1+w_2)}|}{|b_{11}+b_{21}e^{-jw_1}+b_{12}e^{-jw_2}+b_{22}e^{-j(w_1+w_2)}|^2}.$$

The distortion produced by b_{22} decays faster than that produced by b_{21} or b_{12}. A preferred technique should perturb a minimum number of coefficients chosen in the order b_{22}, b_{21}, and b_{12}. The coefficient b_{11} is not perturbed since it is defined to equal 1.0.

Example: For a filter of rotation angle 285° and quantized coefficients as follows:

$\bar{b}_{11}=1.0$	and error	$\beta_{11}=0.0$
$\bar{b}_{21}=0.0$		$\beta_{21}=0.194$
$\bar{b}_{12}=1.0$		$\beta_{12}=-0.180$
$\bar{b}_{22}=-0.5$		$\beta_{22}=0.0144.$

From (24), $\mathrm{Re}(\epsilon)=-0.333$. The filter is unstable. Choose a δ small. Let $\delta=10/100$, $\epsilon=-0.0333$, hence $\mu=-0.3663$.

The choice of the perturbation method is obviously between i) and ii). However, i) is preferred to ii) since the frequency response is least sensitive to a perturbation on b_{22}. The new coefficients by this technique are

$$\tilde{b}_{12}=\bar{b}_{12}(1+\mu)=0.6337$$

$$\tilde{b}_{22}=\bar{b}_{22}(1+\mu)=-0.31685$$

$$\tilde{A}_{z_2}=-\frac{\bar{b}_{11}-\bar{b}_{21}}{\tilde{b}_{12}-\tilde{b}_{22}}=-1.052<-1$$

and hence the filter is now stable. Since coefficient per-

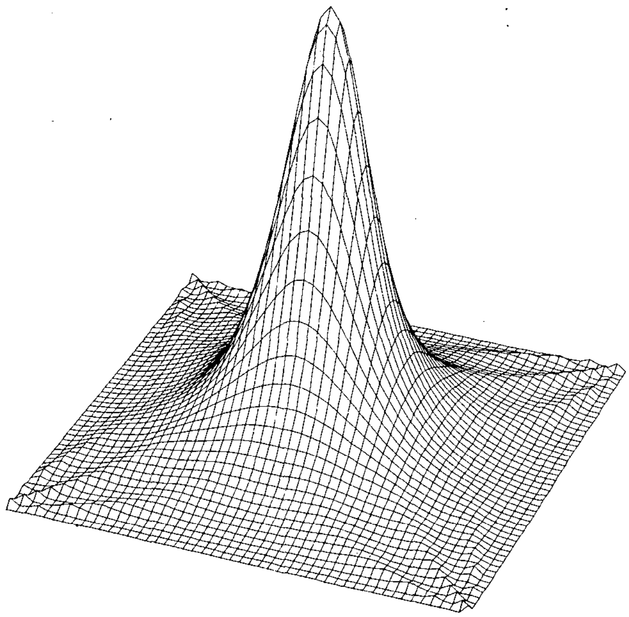

Fig. 5. Frequency response of a two-dimensional digital filter with: 1) exact coefficients (register length used = 32), 2) peak magnitude = 1.000, 3) cutoff frequency = 0.08.

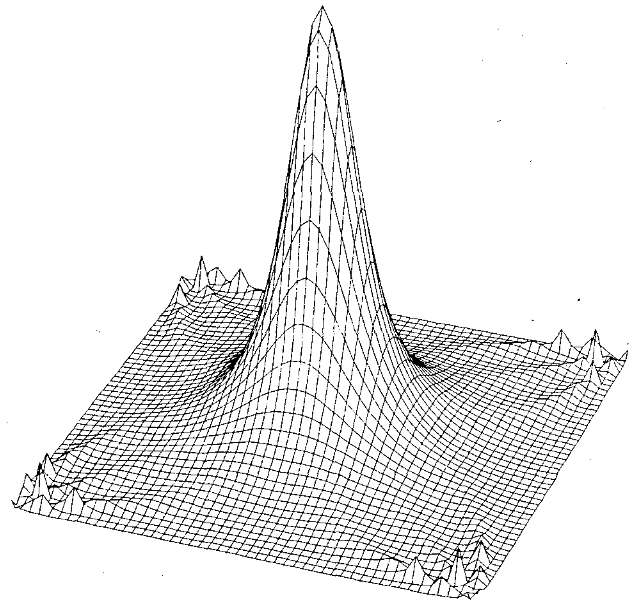

Fig. 7. Frequency response of a two-dimensional digital filter with: 1) register length = 6, 2) peak magnitude = 1.243, 3) cutoff frequency = 0.10.

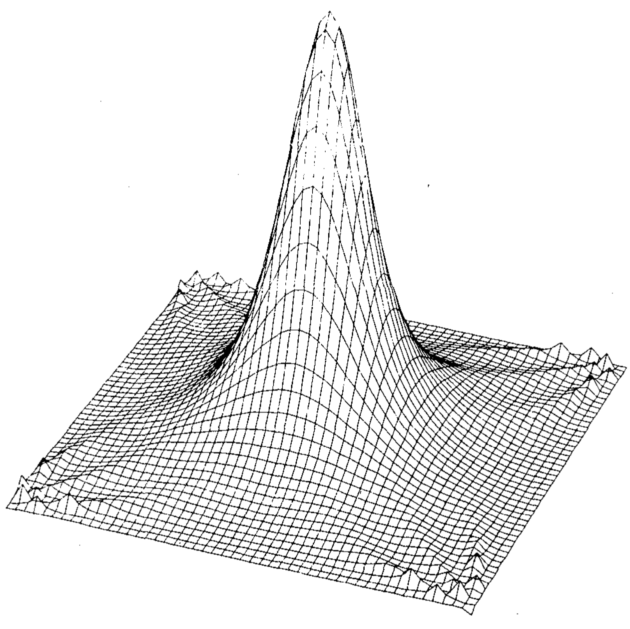

Fig. 6. Frequency response of a two-dimensional digital filter with: 1) register length = 16, 2) peak magnitude = 1.007, 3) cutoff frequency = 0.08.

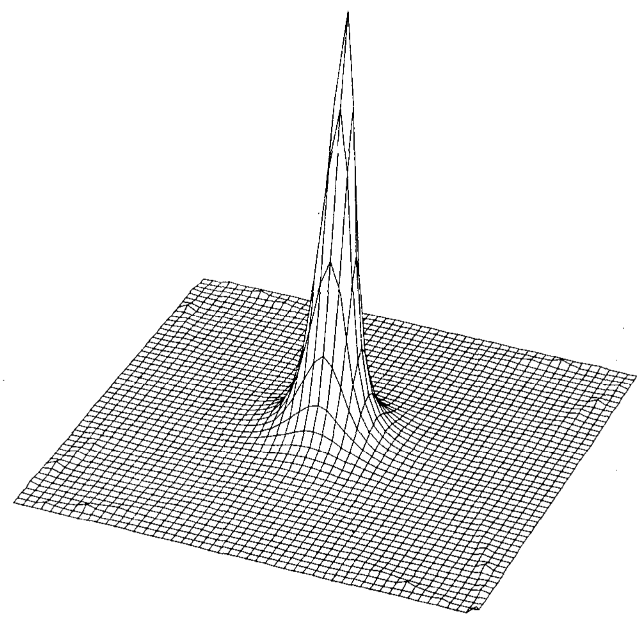

Fig. 8. Frequency response of two-dimensional digital filter with: 1) register length = 4, 2) peak magnitude = 0.598, 3) 3-dB cutoff frequency is very small.

turbation adds distortion to the filter frequency response, perturbations should be kept small.

Fig. 5 shows the frequency response of a filter composed of six low-pass second-order rotated filters cascaded. The rotation angles of the transfer functions are 195°, 225°, 255°, 285°, 315°, and 345°. The first three filters are in the region of instability and cannot be stabilized by the methods discussed above. In this case the input is rotated by 90° in the clockwise direction. A filter with a frequency response rotated by 90° in the counterclockwise direction from the unstable filter is used. The filter is now in the stable region. The output is finally rotated 90° in the counterclockwise direction. This process is equivalent to that of using a filter with a frequency response in the required direction. The technique is discussed in detail in [11]. The filter shown in Fig. 5 has exact coefficients.[3] Figs. 6–8 show the frequency responses obtained by quantizing these coefficients to register lengths of 16, 6, and 4 bits, respectively. Distortions appear in the passband as a change

[3]For exact coefficients the register length used was 32. No additional ... longer register lengths.

in the magnitude and desired cutoff frequency. Distortions also appear as ripples in the stopband. As expected, the distortion increases with a decrease in register length.

VII. Conclusions

The proposed procedure for the design of rotated filters is summarized.

i) Design a marginally stable rotated filter by the method described in [1].
ii) For the special cases discussed in Section V, find the region of guaranteed stability. This region is given by $\{\beta; 270° < \beta_1 < \beta < \beta_2 < 360°\}$, where $\cos\beta_1 + \sin\beta_1 + p_i T/2 = 0$ and $-\cos\beta_2 - \sin\beta_2 + p_i T/2 = 0$, where p_i is the real pole of the one-dimensional continuous filter.
iii) For regions outside this zone of guaranteed stability but within the fourth quadrant compute the stability parameter ϵ. If $\mathrm{Re}(\epsilon) > 0$ the filter is stable. If $\mathrm{Re}(\epsilon) < 0$ the filter is unstable and the perturbation technique is employed to stabilize the filter.

It is more laborious to compute ϵ when floating-point arithmetic is used, however, the filter can be designed inside the region of guaranteed stability, which is easily computed as in ii). Coefficient quantization distorts the transfer function. The distortion increases as the register length is reduced. As observed in Figs. 6–8, there is a change in the magnitude response in the passband, a shift of the cutoff frequency and an increase in the ripples in the stopband. It should be noted that excessively short register lengths may produce a filter with a completely different transfer function. The amount of tolerable distortion depends on the designer's requirements.

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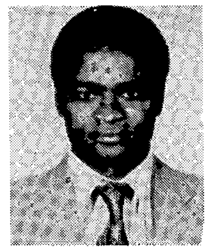

Stanley H. Mneney was born in Monduli, Tanzania. He obtained the B.Sc. (Hons) degree in electrical engineering from the University of Science and Technology, Kumasi, Ghana. He obtained the M.A.Sc. degree in June 79.

He joined the Communications Group in the Department of Electrical Engineering at the University of Toronto in Canada in September 1977. He joined the University of Dar es Salaam in Tanzania as an Assistant Lecturer in 1979. He recently visited Copenhagen, Denmark, working on a solar powered microwave repeater for use in remote areas. His other research interests include digital filters and microprocessor applications in communications. Upon graduation he received the 1976 Charles Deakens Award from the Ghana Institution of Engineers.

Anastasios Venetsanopoulos (S'66–M'69–SM'79) was born in Athens, Greece. He received the B.S. degree in electrical and mechanical engineering from the National Technical University of Athens, Greece, in 1965. He received the M.S., M.Phil., and Ph.D. degrees from Yale University, New Haven, CT, under a U.S. Government Scholarship Grant, in 1966, 1968, and 1969, respectively.

From 1966 to 1968, he was an Assistant in Instruction and from 1968 to 1969, a Research Assistant at Yale University. He joined the Faculty of Engineering of the University of Toronto, Toronto, Ont., Canada in 1968, where he is now a Professor and is the Chairman of the Communications Group. His research interests lie in the areas of digital signal processing and digital communications. He is also a consultant with Electrical Engineering Consociates of Toronto.

Dr. Venetsanopoulos is a member of the Association of Professional Engineers of Ontario, the New York Academy of Sciences, Sigma Xi, the Technical Chamber of Greece, and the American Association for the Advancement of Science. He has served as the Chairman of the IEEE Toronto Section (1977–1979) and Program Chairman of the 1978 International Communications Conference. He is presently the Chairman of the IEEE Central Canada Council. He is a Fellow of EIC and was the Toronto Section Chairman of CSEE (1975–1976) and is presently the Editor of the Canadian Electrical Engineering Journal.

José M. Costa (S'72–M'78) was born in Lérida, Spain. He received the industrial engineering (electronics) degree from the Universidad Politécnica de Barcelona, Spain, and the M.A.Sc. degree in electrical engineering from the University of Toronto, Canada, in 1971 and 1973, respectively. He is presently working part-time towards the Ph.D. degree in the same graduate program. His Master's research was based on the design and implementation of two-dimensional recursive filters and his Ph.D. research is based on the design and implementation of digital tomographic filters for radiographs.

While at the University of Toronto (1971–1976) he was a Teaching and Research Assistant with the Department of Electrical Engineering. During 1977–1978 he held a fellowship in the Department of Communications, Government of Canada, Ottawa, pursuing research on future communication services. He joined Bell-Northern Research Ltd., Ottawa, Canada, in 1978, where has has been working on the planning of new services such as videotex, audio and video retrieval, and image and document communication systems using new technologies such as optical disks and fiber optics.

Mr. Costa is a member of the Society Motion Picture and Television Engineers.

Correction to "The Effects of Quantization Errors on Rotated Filters"

STANLEY H. MNENEY, ANASTASIOS N. VENETSANOPOULOS, AND JOSÉ M. COSTA

In the above paper[1] we found a misprint. In the paragraph following (14), page 998, the signs > and < are exchanged. This paragraph should read:

"The stability parameter ϵ is a complex random variable. When $\mathrm{Re}[\epsilon] > 0$, the transformation $B(z_1, z_2) = 0$ does not cause an overlap of the mapping of the z_1-plane unit circle into the z_2-plane unit circle. The resulting filter is, therefore, stable. When $\mathrm{Re}[\epsilon] < 0$ the filter is unstable. The parameter ϵ can be used to determine the stability status of the filter."

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S. H. Mneney is with the Department of Electrical Engineering, University of Dar Es Salaam, Dar Es Salaam, Tanzania.

A. N. Venetsanopoulos is with the Department of Electrical Engineering, University of Toronto, Toronto, Ontario, Canada.

J. M. Costa is with Bell-Northern Research, Ottawa, Ontario, Canada.

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