

## Reprint

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### Authors' Reply

JOSÉ M. COSTA, ANASTASIOS N. VENETSANOPOULOS,  
AND MARTIN TREFLER

**Abstract**—This reply clarifies that it is possible to change the plane of cut by filtering a tomogram, if the presence of the out-of-focus images is tolerated, because the (projected) images of all the layers are superimposed on the film and any filtering will process all these images simultaneously.

Aprilis<sup>2</sup> has pursued a suggestion made in the paper<sup>1</sup> that in standard tomography we can change the plane of cut by filtering the tomogram. However, the definition of "plane of cut" used in Aprilis<sup>2</sup> is different from that in the paper.<sup>1</sup> In the paper,<sup>1</sup> the plane of cut is simply defined as the layer whose image is in focus (i.e., whose overall transfer function is equal to a constant) and no constraints are put on what happens to the other layers. It was never suggested that the re-

sults of tomographic filtering would be identical to those of a special radiological procedure (see discussion in the paper,<sup>1</sup> p. 82). In Aprilis,<sup>2</sup> a second condition is added to the design of the tomographic filter in an attempt to control the overall transfer function of the other layers. Nevertheless, it is found that the problem is unsolvable because a depth-dependent function appears inside the integral (cf. (4) in Aprilis<sup>2</sup>). This is because in standard tomography and conventional radiology the (projected) images of all the layers are superimposed on the film and any filtering will process all these images simultaneously.

Recognizing that little could be done to eliminate the images of the out-of-focus layers, the characteristics of their overall transfer functions were analyzed in the paper<sup>1</sup> for the case of tomographic filtering of radiographs. It was found that between the plane of cut and the source of X-rays they have low-pass characteristics and between the plane of cut and the film they have high-pass characteristics. As discussed in Aprilis,<sup>2</sup> in the case of tomographic filtering of tomograms, the high-pass characteristics appear to extend over a region around the original plane of cut.

If we accept that the plane of cut is defined as the layer in focus, the open question remains regarding the usefulness of the suggested procedure of changing the plane of cut by means of tomographic filtering. It is not clear if the effects of the high-pass filters are an advantage (because they tend to break away unwanted structures, cf. [1]) or a disadvantage (because they may generate artifacts). Another challenge is how to design better tomographic filters that will minimize the effect of the out-of-focus layers (cf. [2]).

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J. M. Costa is with Bell-Northern Research, Ottawa, Ont., Canada K1Y 4H7.

A. N. Venetsanopoulos is with the Department of Electrical Engineering, University of Toronto, Toronto, Ont., Canada M5S 1A4.

M. Trefler is with the Department of Radiology, Division of Radiological Sciences, School of Medicine, University of Miami, Miami, FL 33101.

<sup>2</sup>E. Aprilis, this issue, p. 68.

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# Correspondence

## Comments on "Digital Tomographic Filtering of Radiographs"

ELENA APRILIS

**Abstract**—In a recent paper,<sup>1</sup> while deriving a mathematical model of standard tomography, the authors state that it is possible to "change the plane of cut by filtering the tomogram." Analyzing the resulting overall transfer function, it appears that the proposed filter transfer function is incapable of doing so. Besides, most likely, there is no filter transfer function, applicable in the suggested procedure, capable of changing the plane of cut. The above mentioned objection is hereby supported by examples.

In their paper,<sup>1</sup> while deriving a mathematical model of standard tomography, the authors state that it is possible to "change the plane of cut by filtering the tomogram."

In particular they claim that "after filtering the tomogram with  $H^{-1}(f_x, f_y)$ " (where  $H(f_x, f_y) = H_t(f_x, f_y, z_t)$  is the transfer function of the layer at a distance  $z_t$  from the film) "the overall transfer function of the layer at a depth  $z_t$  is a constant, thus this layer has become the new plane of cut."

However, the constancy of the overall transfer function in the plane of cut is a necessary condition only, and not a sufficient one, in order to assure that only this plane is evidenced.

In fact, in all previous models of standard tomography (see for example [1]–[5]) the overall transfer function not only assumes a constant value in correspondence to the plane of cut, but it also constitutes a low-pass filter for all other layers (with the cut frequency normally decreasing when the distance from the tomographic plane increases).

In their paper,<sup>1</sup> the authors themselves recognize that "ideally a tomographic filter should have a frequency response such that in combination with the transfer function of the existing system the resulting overall transfer function would be equal to a constant for the tomographic layer and equal to zero everywhere else." It is to be noted nevertheless that in deriving the filter transfer function they only impose the first condition

$$[H_t(f_x, f_y, z_t) \cdot H^{-1}(f_x, f_y)]|_{z_t = z_t} = 1, \quad (1)$$

but not the second one.

As a consequence it may happen that  $H_t(f_x, f_y, z_t)$  satisfies

$$|[H_t(f_x, f_y, z_t) \cdot H^{-1}(f_x, f_y)]|_{z_t \neq z_t}| \geq 1 \quad (2)$$

for at least one  $z_{i0} \neq z_t$ .

In this case the plane at depth  $z_{i0}$  contributes to the final image at least as much as the plane at depth  $z_t$ .

Let us assume, for instance, as it results in [2]–[5], that  $H_t(f_x, f_y, z_t)$  will satisfy

$$|H_{i1}(f_x, f_y, z_{i1}) \cdot H_{i2}^{-1}(f_x, f_y, z_{i2})| \geq 1 \quad (3)$$

for "almost all"  $(f_x, f_y)$  if

$$|z_{i1} - \Delta_2| \leq |z_{i2} - \Delta_2| \quad (3')$$

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The author was with the Department of Electrical Engineering, University of Padua, Padua, Italy. She is now at Via Montebelluna 37, 33170 Pordenone, Italy.

<sup>1</sup>J. M. Costa, A. N. Venetsanopoulos, and M. Trefler, *IEEE Trans. Med. Imaging*, vol. MI-2, pp. 76–88, June 1983.

where  $\Delta_2$  is the depth of the original plane of cut, being the origin of the coordinates in the plane of the film.

After filtering the tomogram with  $H^{-1}(f_x, f_y)$  we do not obtain the image of a layer but the one of a slice at least  $2|z_t - \Delta_2|$  thick, with the old plane of cut ( $z_i = \Delta_2$ ) in the middle. Moreover, some frequency components of the layers  $|z_i - \Delta_2| \leq |z_t - \Delta_2|$  are amplified (more precisely, the frequencies  $(f_x, f_y)$  in correspondence to which the left-side term in (3) is higher than one).

On the other hand, if the second condition (zero values, or at least low pass for  $z_i \neq z_t$ ) is imposed, some degree of dependence of the filter transfer function on  $z_t$  is introduced. As a consequence, the term  $\bar{H}^{-1}(f_x, f_y, z_t)$  cannot be moved out of the sign of integral in the equation

$$\int_0^d H_t(f_x, f_y, z_t) \cdot \bar{H}^{-1}(f_x, f_y, z_t) \cdot F_\mu(f_x, f_y, z_t) dz_t \quad (4)$$

as indicated in (20) of the paper,<sup>1</sup> where, at variance with the present situation,  $H^{-1}(f_x, f_y)$  was not dependent on  $z_t$  once  $z_t$  had been fixed.

Therefore, both sides of the equality

$$G(f_x, f_y) = I_B \delta(f_x, f_y) - \int_0^d H_t(f_x, f_y, z_t) \cdot F_\mu(f_x, f_y, z_t) dz_t \quad (5)$$

cannot be divided by the same function  $\bar{H}(f_x, f_y, z_t)$ .

In conclusion, it appears that the suggested procedure of tomogram filtering is not able to change the plane of cut. Some examples in the following will support this point; it is important to note that fixing the shape of function  $H_t(f_x, f_y, z_t)$  is not restrictive, as similar results would be obtained with all the functions for which condition (2) holds.

Let us suppose, to simplify, that  $H_t(f_x, f_y, z_t) = H_t(f_y, z_t)$  (linear tomography in direction  $y$ ) and that the origin of the coordinates lies in the tomographic plane.

Assume, as in [3], [4], that  $H_t(\omega_y, z_t)$  is given by

$$H_t(\omega_y, z_t) = \frac{\sin(K\omega_y z_t)}{K\omega_y z_t} \quad (6)$$

where  $K$  is a constant depending on the system geometry.

Let us examine the statement

$$\left| \frac{H_{i1}(\omega_y, z_{i1})}{H_{i2}(\omega_y, z_{i2})} \right| = \left| \frac{\text{sinc}(K\omega_y z_{i1})}{\text{sinc}(K\omega_y z_{i2})} \right| \geq 1 \quad (7)$$

when  $|z_{i1}| < |z_{i2}|$  [cf. (3)].

First, observe that

$$H_{i1}(\omega_y, z_{i1}) = 0 \quad \text{for } \omega_y = n\pi/Kz_{i1} = n\omega_{y1}$$

$$H_{i2}(\omega_y, z_{i2}) = 0 \quad \text{for } \omega_y = n\pi/Kz_{i2} = n\omega_{y2}$$

where  $n$  is an integer different from zero.

Let us write  $|z_{i1}| = |z_{i2}|/m$  where  $m$  is a number higher than 1, then  $\omega_{y1} = m\omega_{y2}$ .

If  $m$  is an integer, then relation (7) is satisfied for all  $\omega_y$  (see

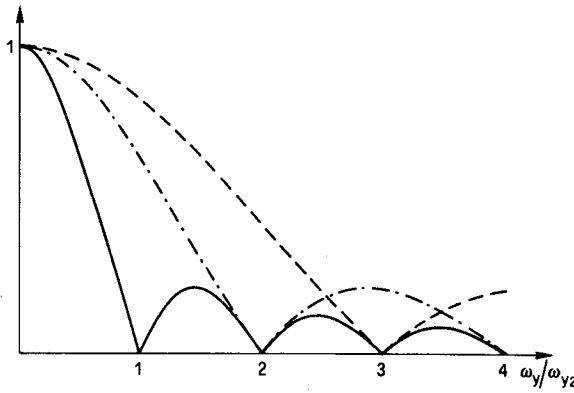


Fig. 1. Plot of  $\text{sinc}(K\omega_y z_i)$ : —  $|z_i| = |z_{i2}|$ , - - -  $|z_i| = |z_{i2}|/2$ , - · -  $|z_i| = |z_{i2}|/3$ .

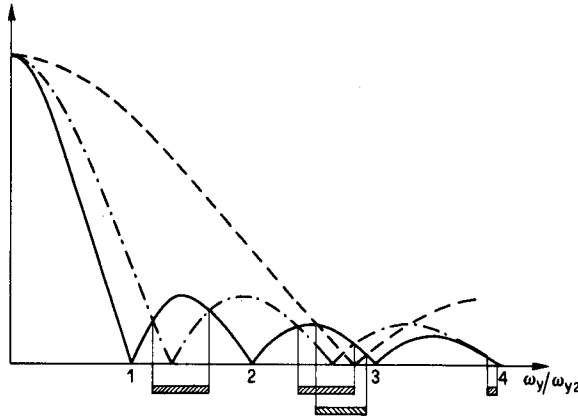


Fig. 2. Plot of  $\text{sinc}(K\omega_y z_i)$ : —  $|z_i| = |z_{i2}|$ , - - -  $|z_i| = |z_{i2}|/1.3$ , - · -  $|z_i| = |z_{i2}|/2\sqrt{2}$ .

Fig. 1). If  $m$  is a real number, then relation (7) is satisfied for "almost all"  $\omega_y$  (see Fig. 2).

As a result, after application of the filtering procedure, the overall transfer function could be acceptable for layers at depth  $|z_i| > |z_t|$  but it assumes values very far from the ideal one for layers at depth  $|z_i| \leq |z_t|$ . In fact, all the planes with  $|z_i| \leq |z_t|$  would contribute to the final image with a term, the value of which is higher for the planes which are closer to the old plane of cut.

To actually change the tomographic plane, the original transfer function of the system should be multiplied by a function  $P(\omega_y, z_i)$  such that

$$\text{sinc}(K\omega_y z_i) P(\omega_y, z_i) = \text{sinc}[K\omega_y(z_i - z_t)]$$

i.e., by

$$P(\omega_y, z_i) = \frac{\text{sinc}[K\omega_y(z_i - z_t)]}{\text{sinc}(K\omega_y z_i)}$$

Unfortunately, this function is dependent on  $z_i$ , and is therefore useless as previously pointed out.

Consider now the expression (17) of the paper<sup>1</sup>

$$H_i(f_x, f_y, z_i) = K_i^2 \iint I_0(K_i x, K_i y) e^{-j2\pi(f_x x + f_y y)} dx dy.$$

$H_i(f_x, f_y, z_i)$  depends on  $z_i$  through  $K_i$

$$K_i = \frac{z_i - d}{z_i - \Delta_2} \frac{\Delta_1}{d}$$

where

$$d = \Delta_1 + \Delta_2$$

$\Delta_1$  is the distance between the original plane of cut and the plane of the source

$\Delta_2$  is the distance between the original plane of cut and the plane of the film.

We can write

$$K'_i = K_i \frac{d}{\Delta_1} = \frac{z_i - d}{z_i - \Delta_2} = 1 - \frac{\Delta_1}{z_i - \Delta_2}$$

if  $z_i < \Delta_2$  then

$$K'_i = 1 + \frac{\Delta_1}{\Delta_2 - z_i} > 1$$

if  $z_i > \Delta_2$  then

$$K'_i = 1 - \frac{\Delta_1}{z_i - \Delta_2} < 1.$$

Assuming the intensity of the X-ray source  $I_0(x_0, y_0)$  not depending on  $(x_0, y_0)$  (coordinates in the plane of the source), we obtain

$$H_i(f_x, f_y, z_i) = K_i^2 I_0 \bar{H}(f_x, f_y).$$

Let us suppose that the new plane of cut should be the plane at depth  $z_t$ . We have

$$|H_i(f_x, f_y, z_i) H_t^{-1}(f_x, f_y, z_t)| = \frac{K_i^2}{K_t^2} = \frac{K_i'^2}{K_t'^2}.$$

Therefore, if  $z_t < \Delta_2$

$$|H_i(f_x, f_y, z_i) H_t^{-1}(f_x, f_y, z_t)| = \begin{cases} \geq 1 & \text{for } z_i < \Delta_2 \text{ and } |\Delta_2 - z_i| \leq |\Delta_2 - z_t| \\ < 1 & \text{for } z_i > \Delta_2 \text{ or } z_i < \Delta_2 \text{ and } |\Delta_2 - z_i| > |\Delta_2 - z_t| \end{cases}$$

if  $z_t > \Delta_2$

$$|H_i(f_x, f_y, z_i) H_t^{-1}(f_x, f_y, z_t)| = \begin{cases} > 1 & \text{for } z_i < \Delta_2 \text{ or } z_i > \Delta_2 \text{ and } |\Delta_2 - z_i| > |\Delta_2 - z_t| \\ \leq 1 & \text{for } z_i > \Delta_2 \text{ and } |\Delta_2 - z_i| \leq |\Delta_2 - z_t|. \end{cases}$$

Considering that the overall transfer function for the "new plane of cut" is  $|H_t(f_x, f_y, z_t) H_t^{-1}(f_x, f_y, z_t)| = 1$ , we can conclude the following.

When  $z_t < \Delta_2$  is the final image, there are contributions from all the layers between the old plane of cut and the new one.

When  $z_t > \Delta_2$  there are contributions from all the layers between the old plane of cut and the plane of the film and between the new plane of cut and the plane of the source.

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