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## DIGITAL TOMOGRAPHIC RESTORATION OF RADIOGRAPHS\*

J.M. Costa and A.N. Venetsanopoulos<sup>†</sup>Summary

Conventional radiographs are two-dimensional projections of three-dimensional objects. Taking advantage of the finite size of the X-ray source and the divergent nature of the X-ray beam, a radiograph can be restored so that the image of a particular layer is deblurred, while the others are not. We refer to this method as a tomographic filtration process and it can be implemented using digital inverse filtering techniques.

Theoretical and practical evaluations of performance of tomographic filters have shown that while the results cannot achieve the quality of those obtained by special radiographic procedures, such as standard tomography or computerized tomography, they can improve the usefulness of conventional radiographs with respect to depth information. This can be accomplished by making some structures more visible or helping to interpret those structures, which are readily seen, while at the same time improving the speed and reducing the cost and the problems related to patient overdose.

1. Introduction

One important application of two-dimensional digital filtering is found in the processing of radiographs. The work done so far in this area has concentrated in the aspects of enhancement, restoration, and pre-processing for pattern recognition. It was usually assumed that the objects being X-rayed are two-dimensional.

Radiographs are two-dimensional representations of three-dimensional objects. Using conventional X-ray imaging techniques a shadow view of the body under examination is produced, which does not contain explicit information of the depth of details and structures. This drawback has been overcome partially by special radiographic techniques such as standard tomography [1] and three-dimensional radiography or computerized tomography [2]. With these techniques care must be taken to avoid extreme complexity, patient overdose, excessive cost, and time consumption.

This research project is concerned with the problem of retrieving three-dimensional information from radiographs, without altering the specifications or operating conditions of the X-ray system. Taking advantage of the finite size of the X-ray source and the divergent nature of the X-ray beam, a radiograph can be restored so that the image of a particular layer is deblurred while the others are not. We refer to this method as a tomographic filtration process.

It can be shown that due to the finite size of the X-ray source, the image of each layer of a three-dimensional object exposed to X-rays is blurred to an extent which depends on the distance from that layer to the

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source of X-rays and to the film plane. The extent of the blur is directly proportional to the layer's magnification and to the size of the source of X-rays.

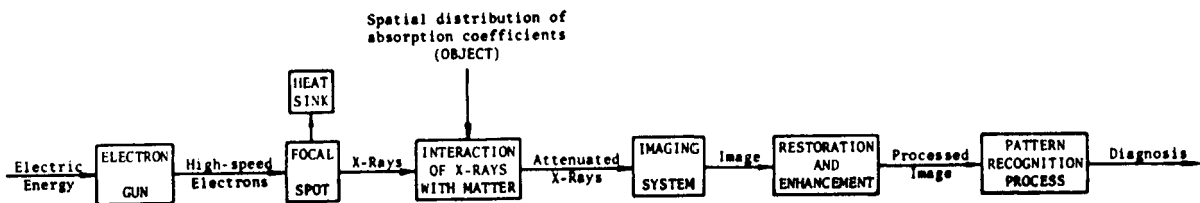
To determine what characteristics a tomographic filter should have, a model for the radiologic process from the viewpoint of communication system theory was developed. In the ideal case a tomographic filter has the characteristics of an inverse filter, scaled according to the magnification of the layer concerned.

## 2. A Model for the Radiologic Process

Before we attempt any improvement of radiographs we must study the characteristics of the image formation process to find out what are the degradations which should be corrected, especially those which depend on the depth.

The radiologic process consists of a sequence of transformations intimately related in that the result of one forms the input to the next [3]. The degradations introduced at each stage of the radiologic process have been studied in great detail from the viewpoint of image quality.

In Figure 1 we propose a block diagram model for the radiologic process. In general, block inputs and outputs are two-dimensional functions representing distributions of intensities.



(a)

Fig. 1. Block Diagram for the Radiologic Process

The first two blocks represent the generation of X-rays. When the beam of high-speed electrons is stopped by a tungsten target some of its kinetic energy is converted into electromagnetic energy. A very large amount of heat is also produced. The region in the target where the X-rays and heat are produced is called the focal spot. The angle formed by the target surface and the direction of the centre X-ray is referred to as the target angle.

Many studies have been published about the characteristics of focal spots in X-ray tubes. The shape and size of typical focal spots have been determined as well as their modulation transfer functions (MTF's), both theoretically and experimentally. The MTF is the magnitude of the fourier transform of the point spread function (PSF) or impulse response of the focal spot. The PSF is defined here as a pin-hole image of the focal spot.

The interaction of X-rays with matter may be modelled by

$$I(x) = I(0) \exp \left\{ - \int \mu(x) dx \right\} \quad (1)$$

where  $I(x)$  is the intensity of a narrow X-ray beam as a function of the distance  $x$  in the direction of propagation and  $\mu(x)$  is a total attenuation coefficient.

X-rays propagate in straight lines. This fact controls the size, shape, and position on the radiographic film of the shadow or image of the various structures of the object being exposed.

Due to the diverging nature of the X-rays emitted by the focal spot, the size of the shadow is different from that of the object casting the shadow. The ratio of the size of the image to that of the object is called the magnification. For a three-dimensional object, the magnification is a constant in a layer parallel to the film plane. If we denote by  $d_1$  the distance from the focal spot to the  $i^{\text{th}}$  layer and by  $d_2$  the distance from that layer to the film plane, the magnification for that layer is given by

$$m_i = (d_1 + d_2)/d_1 \quad (2)$$

The effect of a focal spot of finite size is obtained by integrating (1) for all the elemental sources which compose the focal spot. If at the same time we approximate the integral in the exponential in (1) by a summation, it results in a product of exponentials as follows:

$$I(x,y) = \iint_{\text{F.S.}} I_0(x_0,y_0) \prod_i \exp\{-\mu_i(x_i,y_i)\Delta\ell\} dx_0 dy_0 \quad (3)$$

where  $I(x,y)$  and  $I_0(x_0,y_0)$  are the X-ray intensity distribution over the film and the focal spot (F.S.), respectively,  $\mu_i(x_i,y_i)$  is the distribution of absorption coefficients in the  $i^{\text{th}}$  layer of the object, the coordinates  $(x_i,y_i)$  correspond to the intersection of the X-ray from  $(x_0,y_0)$  to  $(x,y)$  with the  $i^{\text{th}}$  layer, and  $\Delta\ell$  is the thickness traversed by that X-ray in the  $i^{\text{th}}$  layer.

X-rays, at the energy levels used in radiology, have very poor capability for impressing film. Consequently, additional devices, such as image intensifiers or screens, must be used. The function of these imaging systems is to introduce a large number of light photons in the device output for each X-ray photon in the input.

The radiographic image is characterized by many unwanted effects in the X-ray exposure process such as: noise, low-contrast, distortions, blur, etc. The restoration of radiographs attempts to correct some of these perturbations. In the next section we study a type of restoration, which is tomographic, because it takes into account the depth. On the other hand the fidelity criterion of enhancement is not attempting better object representation; but provides additional information or insight into some factor concerning the pre-enhanced image. The enhancement of radiographs may also be tomographic [4], but this will not be discussed here.

The final stage in the radiologic process is the pattern recognition, which may be performed by an automated process, a radiologist, or both in combination.

### 3. A Tomographic Filtration Process

When the object which can be visualized as a three-dimensional distribution of absorption coefficients is projected and transformed into a light image, several types of degradations are introduced, as discussed in the previous section. There is only one however which is depth dependent, namely the one due to the finite size of the focal spot. Indeed, if the source of X-rays was a perfect point source no depth information recovery would be at all possible from a single radiograph.

Therefore our approach to the tomographic restoration of radiographs utilizes the depth-dependent focal-spot blur. The tomographic effect will be better understood by comparing this filtration process with standard tomography.

Standard tomographic techniques produce a tomogram by moving a point-like X-ray source and the recording film in a coupled manner, so that during the exposure the parts of the object lying in one specific plane parallel to the film plane are always projected on the same place on the film [1]. The X-ray shadows of the other parts of the object will move in relation to the film. Thus only a layer of a few millimeters of thickness in a predefined depth of the body is imaged sharply, whereas structures above and below this layer are blurred. The layer whose image is in focus is referred to as the plane of cut or tomographic layer.

A tomographic filtration process (TFP) produces a focussing effect similar to that of standard tomography; but with no moving parts. In a TFP, instead of moving the X-ray tube, the finite size of the focal spot is used to advantage and instead of moving the film, a filter is used to process a conventional radiograph. Indeed, by applying superposition we can image an equivalent source of X-rays by moving a point source all over the region of the actual source. The movement of this point source is analogous to the movement of an X-ray tube in standard tomography. Since in conventional radiography the film does not move, the images of all the layers are blurred. Therefore, in order to convert a radiograph into a tomogram we will pass the radiographic image through a filter that will produce on the image of the tomographic layer an effect equivalent to that produced by the motion of the film in standard tomography. Since a typical size for the focal spot is of the order of 2 mm., while the movement of an X-ray source in standard tomography is of the order of 500 mm., we infer that the results of a TFP cannot be as good as those of standard tomography. Nevertheless, the purpose of a TFP is to improve a conventional radiograph, with respect to depth information, and not to produce a complete and accurate three-dimensional reconstruction, for which better techniques exist [2].

The mathematical characteristics of tomographic filters have been derived in [4]. It has been shown that the transfer function of the tomographic restoration filter is the inverse of the transfer function for this layer. It has also been shown [4] that the system impulse response is given by the intensity distribution of the X-ray source, spatially scaled in each direction by the factor  $-d_1/d_2$ , where  $d_1$  and  $d_2$  are the distances from the tomographic layer to the source and to the film, respectively.

Since the gain of an ideal inverse filter can become very high it is necessary to hard-limit the magnitude response of the tomographic filter in order to prevent the amplification of noise.

Suppose that  $H = a + j b$  is the system transfer function for the tomographic layer.  $H$ ,  $a$ , and  $b$  are functions of the spatial frequency  $(f_x, f_y)$  and  $j$  is the imaginary operator.

The magnitude response of the inverse filter is

$$\left| \frac{1}{H} \right| = \frac{1}{\sqrt{a^2 + b^2}} \quad (4)$$

and the phase response

$$\angle \frac{1}{H} = \arctan \left( \frac{-b}{a} \right) \quad (5)$$

The transfer function of the inverse filter with hard-limited magnitude response is expressed as:

$$H^{-1} = c + j d \quad (5)$$

with the constraint

$$\sqrt{c^2 + d^2} \leq h_\ell \quad (7)$$

where  $h_\ell$  is the hard limit imposed on the magnitude response and  $c$  and  $d$  are given by

$$c = \frac{a}{a^2 + b^2} \quad \text{and} \quad d = \frac{-b}{a^2 + b^2} \quad \text{for} \quad \frac{1}{\sqrt{a^2 + b^2}} \leq h_\ell \quad (8a)$$

$$c = \frac{ah_\ell}{\sqrt{a^2 + b^2}} \quad \text{and} \quad d = \frac{-bh_\ell}{\sqrt{a^2 + b^2}} \quad \text{for} \quad \frac{1}{\sqrt{a^2 + b^2}} > h_\ell \quad (8b)$$

$$c = h_\ell \quad \text{and} \quad d = 0 \quad \text{for} \quad a^2 + b^2 = 0 \quad (8c)$$

Equations (8a,b,c) are consistent with (5) and (7); that is the phase response is preserved and the dynamic range of the magnitude response can be controlled with the parameter  $h_\ell$  to prevent noise amplification and/or overflow of computer registers. In a digital computer all these operations can be performed in a straightforward manner. It should be noted that under the transformations (8) a real PSF remains real, and an even PSF remains even.

In many cases it is convenient to assume a Gaussian shape for the system transfer function. The magnitude response of the tomographic filter then becomes an inverse Gaussian curve, which is readily implemented in a digital computer.

Since the system transfer function usually has a low-pass characteristic, the inverse filter has a high-pass characteristic. Therefore it is convenient to cascade the inverse filter with a low-pass filter to reduce the noise at high frequencies where the gain of the inverse filter is greatest. The choice of the cutoff frequency of the low-pass filter is a trade-off between the desired resolution and noise.

An analysis in the frequency domain of the performance of tomographic filters has shown [4], that in the low frequency region a tomographic filter acts as a lowpass filter for layers below the plane of cut and as a highpass filter for layers above it.

#### 4. Design and Realization of Digital Tomographic Filters

Once the mathematical characteristics of a tomographic filter have been determined, the implementation can be done optically or digitally. The digital processing of two-dimensional signals offers the advantages characteristic of digital computers, such as flexibility and accuracy and for those reasons it was preferred over optical processing.

The data which must first be collected are the characteristics of the radiologic system, a radiograph or set of radiographs, and information on

the geometries of the set-ups. The radiologic system can be characterized by a pin-hole image of the X-ray source and the geometry of the set-up. By sampling this pin-hole image and interpolating, the impulse responses for different layers can be determined. The radiographs must also be sampled for processing in a digital computer. Details of the digitization process and the scaling can be found in [4].

While in our tests [4] we used separable two-dimensional digital filters to improve on computer memory usage, these filtering techniques can be also applied with non-separable filters. For simplicity the example in Fig. 2 is one-dimensional.

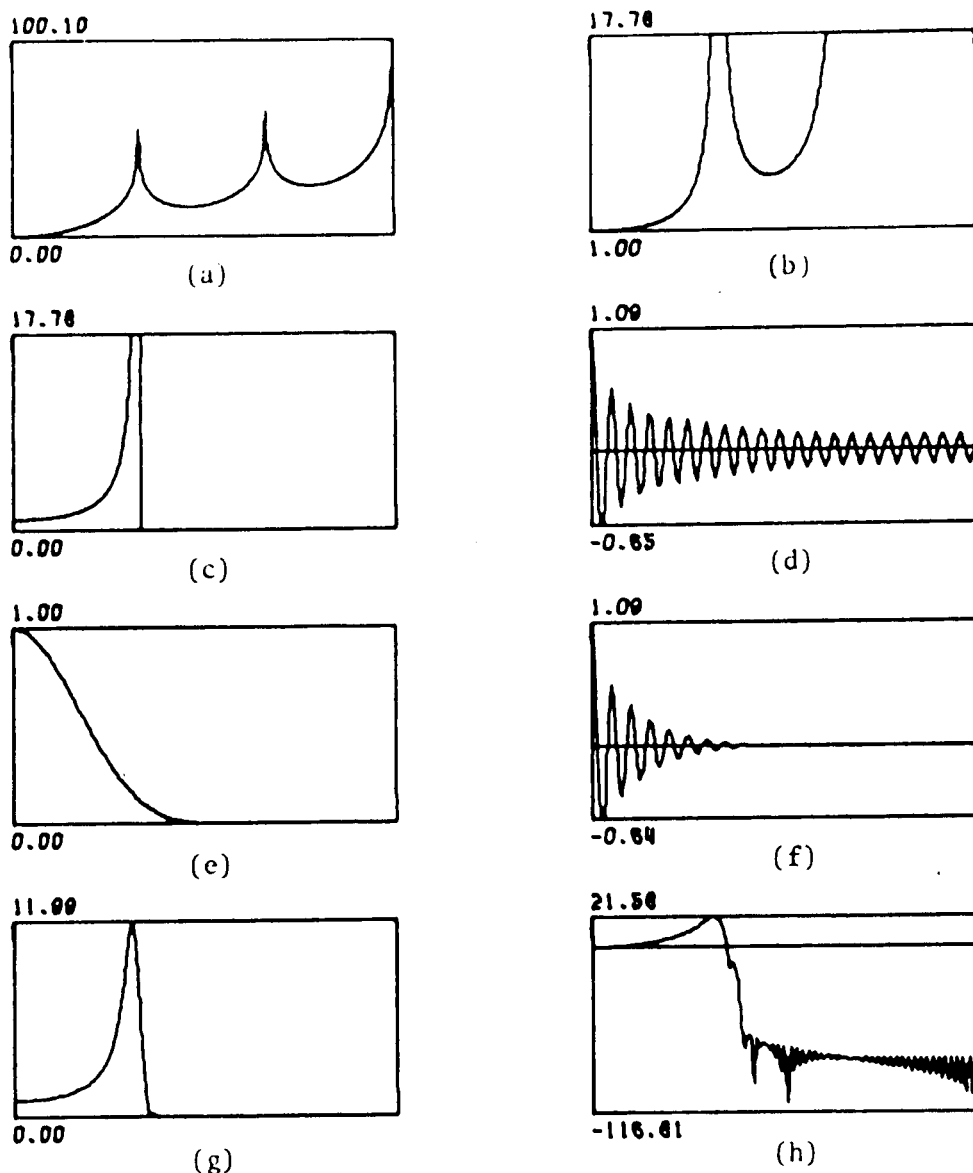


Figure 2. Plots of relevant functions in the design of a digital tomographic filter using the windowing technique. (a) Ideal inverse filter (in dB). (b) Inverse filter with hard-limited magnitude response. (c) The filter in (b) cascaded with an ideal low-pass filter. (d) Impulse response of the filter in (c). (e) Kaiser window with  $\beta=9$ . (f) Windowed impulse response. (g) Magnitude response of the tomographic filter. (h) Same as (g) in dB.



Once the impulse response has been determined for a layer of interest, the transfer function of the inverse filter is calculated (Fig. 2(a)) and the coefficients are adjusted so that the gain is never higher than a certain limit (Fig. 2(b)). Then the inverse filter is cascaded with a lowpass filter to reduce the high frequency noise (Fig. 2(c)). The ideal impulse response of this filter is shown in Fig. 2(d). Due to the current computational constraints in the two-dimensional digital filter optimum design techniques, the windowing method is the only one, which can be used to approximate a completely arbitrary complex frequency response [5]. Different types of windows provide different tradeoffs between the resolution and ripple in the overall frequency response. We have chosen to use a Kaiser window [6] (Fig. 2(e)), which contains a parameter  $\beta$  that controls the frequency response tradeoff between resolution and ripple. In this application we have used a high  $\beta$ , such as  $\beta=9$ , in order to obtain low ripples in the stopband and a smooth transition band. The windowed impulse response is shown in Fig. 2(f) and the resulting magnitude response in Fig. 2(g)-(h).

Fast convolution techniques are used in the filtering implementation. A portion of the digitized radiograph is chosen and multiplied by a two-dimensional cosine taper data window to reduce the effects of leakage. It is then Fourier transformed with the two-dimensional FFT. We found the size  $256 \times 256$  to give a good tradeoff between resolution and cost. The transform of the radiograph and the filter coefficients are complex multiplied point-by-point. The result is inverse transformed, stored on magnetic tape and displayed.

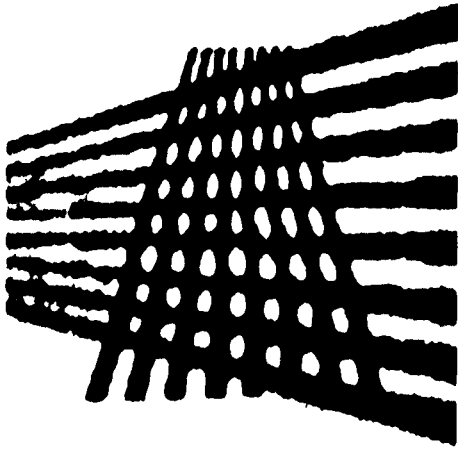
The previously described technique was tested with both computer-simulated radiographs of test patterns and actual radiographs. An example is shown in Fig. 3.

The basic test pattern which we used for the computer simulation of radiographs consists of a series of converging bars with 50% absorption. The convergence of the bars provides different spacings among them and therefore the effect of filtering on different frequencies can be examined. In order to have a three dimensional object two basic test patterns were superimposed at two different levels with the bars oriented perpendicularly. In this simulation the focal spot to film distance was 1000 mm. and the distances from the two layers in the object to the film were 400 mm. and 600 mm. (see [4] for more details). The focal spot had a Gaussian intensity distribution defined by

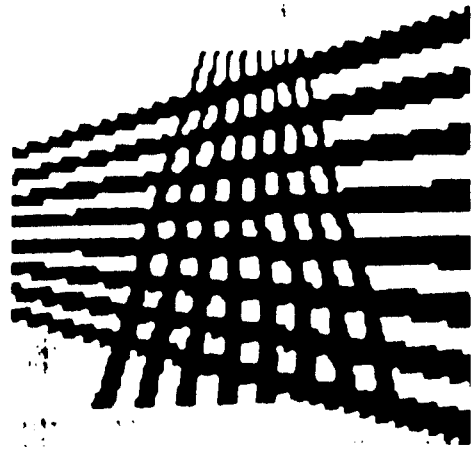
$$I_0(x_0, y_0) = \exp\left(-2.0(x_0^2 + y_0^2)\right) \text{ for } -1.4 < x_0, y_0 < 1.4 \text{ mm.}$$

The result of the simulation is shown in Figure 3(a). For comparison purposes and in order to have an ideal image as a reference Figure 3(b) shows the simulation of an x-ray image of the same object obtained with a punctual focal spot. Figures 3(c) and 3(d) show magnifications of the central part of the radiographs in Figures 3(a) and 3(b), respectively, multiplied by a two-dimensional cosine taper data window.

Figures 3(b) and 3(d) show a block-like structure not visible in Figures 3(a) and 3(c) because it is smeared out by the blur. This block-like nature is due to the magnification of the sampling intervals in the object when they are projected in the film. This is an advantage here because it permits the evaluation of the recovery of small details with tomographic filters.



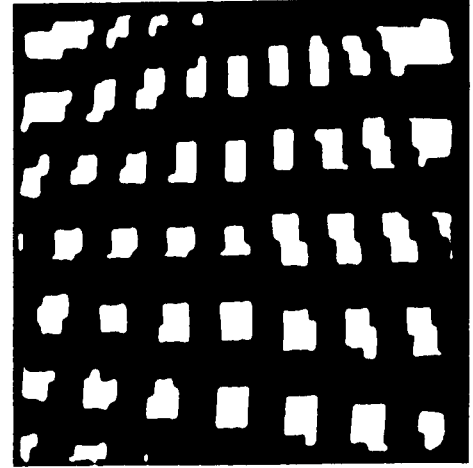
(a)



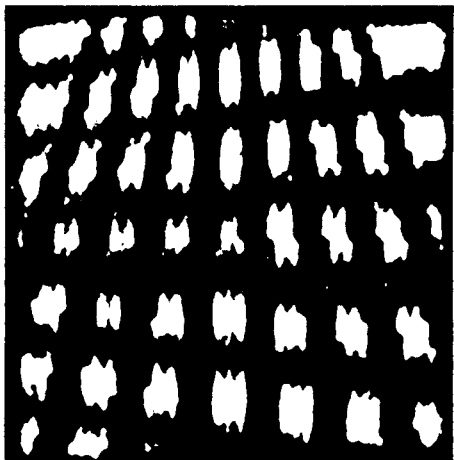
(b)



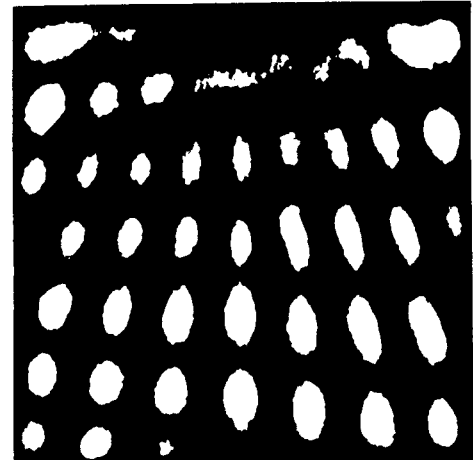
(c)



(d)



(e)



(f)

Fig. 3. (a) - (f) Computer Simulated Radiographs and Examples of Tomographic Filtration (see text).

Tomographic filters were designed as discussed previously. Two examples of filtering Figure 3(c) are shown in Figures 3(e) and 3(f). In Figure 3(e) we are restoring the sharpness of the layer with the lower magnification ("the vertically-oriented bars") and in Figure 3(f) we are restoring the layer with higher magnification ("the horizontally oriented bars"). Since the layers closer to the film have an impulse response of smaller extent, restoration is easier for these layers.

## 5. Conclusions

In this paper we have summarized a technique for the synthesis of a class of digital filters for tomographic deblurring of radiographs. Theoretical and practical evaluations [4] of the performance of tomographic filters have shown that while the results cannot achieve the quality of those of standard tomography or three-dimensional X-ray reconstruction techniques, they can be easily implemented, they do not require any special X-ray hardware, through facilities for optical or digital signal processing are necessary and they eliminate problems related to patient overdose and excessive cost reducing complexity and improving the speed.

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