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Tomographic filters for digital radiography

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Abstract

Conventional radiographs (digitized or not) do not provide information about the depths of details and structures because they are two-dimensional projections of three-dimensional bodies. Taking advantage of the finite size of the X-ray source and the divergent nature of the X-ray beam, a radiograph can be processed by two-dimensional digital filtering techniques, so that the image of a particular layer is improved, while the others are degraded. This technique is referred to as a Tomographic Filtration Process (TFP). This paper explains the mathematical and physical foundations of the method and the engineering considerations in the design and realization of tomographic filters. Theoretical comparisons between conventional radiography, standard tomography and tomographic filtering are discussed in terms of the thickness of the tomographic layer, the rate of change of the Modulation Transfer Function (MTF) and the signal to noise ratios. Finally, experimental results are shown to demonstrate the effect of tomographic filtering at different depths.

Introduction

The problem of imaging three-dimensional bodies and obtaining information about the depth of details and structures has been recognized for a long time. As early as 1916 special radiographic procedures were invented to obtain clear images of certain parts of an object by blurring images from other parts [1]. These procedures will be referred to throughout this paper as standard tomography. More recently, significant advancements have been made in computer-assisted tomography (CAT) and coded aperture imaging [2]. One of the remaining challenges is to improve the diagnostic value of the billions of radiographs being produced in hospitals every year using conventional radiography equipment. Conventional radiography does not include special techniques that will highlight single layers in the body being imaged. Enhancement and restoration techniques have had limited practical application to the processing of radiographs [2]. With digital radiography, techniques that had only been used in lab experiments could now be applied much more readily. However, little work has been published on the problem of recovering three-dimensional information from a single radiograph (cf. [3]).

The purpose of this paper is to describe a procedure for improving three-dimensional information in a radiograph using digital techniques. This method will be referred to as tomographic filtering or a tomographic filtration process (TFP).

Before any improvement of radiographs can be attempted, it is necessary to study the characteristics of the image formation process to find out what are the depth-dependent features. To date, almost all theory of conventional radiography has dealt with two-dimensional objects (cf. [4]). The very nature of the radiologic process, however, forces one to consider three-dimensional objects in all imaging problems. The radiographic process consists of a sequence of transformations intimately related in that the result of one forms the input to the next [5]. An analysis of these transformations has shown that the only effects that could be useful for obtaining three-dimensional information are due to the finite size of the focal spot and the diverging nature of the X-ray beam [6]. Indeed, due to the finite size of the focal spot, there is a blur associated with the image of each layer, which is depth-dependent. This suggests that there could be some kind of selective restoration of each layer's image (tomographic restoration). Also, the diverging nature of the X-ray beam has the effect of magnifying each layer in the film, so that the spectra of the layers are scaled differently. This suggests that selective enhancement of the layers could be realized by means of spectral shaping filters (tomographic enhancement).

Mathematical foundation of tomographic filtering

This approach to tomographic filtering of radiographs uses the depth-dependent focal-spot blur. A tomographic filtration process (TFP) will be understood better by comparing it with standard tomography.

Standard tomographic techniques produce a tomogram by moving a point-like X-ray source and the recording film in a coupled manner, so that during the exposure only the parts of the body lying in one specific plane parallel to the film plane are always projected on the same place on the film, while the others are blurred [1]. The layer whose image is in focus is referred to as the plane of cut or tomographic layer.

A tomographic filtration process (TFP) should produce a focusing effect similar to that of standard tomography; but with no moving parts. In a TFP, instead of moving the X-ray tube, the finite size of the focal spot is used to advantage and instead of moving the film, a filter is used to process a conventional radiograph. Indeed, by applying superposition an equivalent source of X-rays could be conceived by moving a hypothetical point source all over the region of the actual source. The movement of this point source is analogous to the movement of an X-ray tube in standard tomography. Since in conventional radiography the film does not move, the images of all the layers are blurred. Therefore, in order to convert a radiograph into a tomogram the radiographic image is processed by a filter that will produce an effect equivalent to that produced by the motion of the film in standard tomography.

To derive the transfer function of such a filter, the equations of image formation in radiography had to be derived for three-dimensional objects. To make the results more general and allow comparisons between systems, the model of conventional radiography was derived as a special case of standard tomography. This derivation was motivated by that in [7]. Some of the constraints in [7] were removed, namely, the linear movement of a constant-intensity X-ray source, while others relevant to this application were added, namely, small displacements of the X-ray source. Nevertheless, none of these constraints imply a lack of generality in the derivation.

Consider the diagram of standard tomography shown in Figure 1. The reference coordinate systems shown in Figure 1 are self-explanatory: x_o,y_o is the film plane, x_t,y_t is the plane of cut, x_i,y_i is any layer in the object at depth z_i (its distance to the film plane), x_f,y_f is the (moving) film and x,y is the (fixed) plane containing the film. In this model, the X-ray point source (X) can move anywhere in the plane x_o,y_o parallel to the film and the intensity during this trajectory is given by $I_o(x_o,y_o)$, which will be referred to as the exposure function. In standard tomography the film also moves in synchronism with the X-ray source to keep the desired plane of cut x_t,y_t in focus.

It is not possible to reproduce here all the details of the derivation, which can be found in [6]. However, it is important to note the approximations made:

1) The X-ray intensity from (x_o,y_o) to (x_f,y_f) is independent of (x_f,y_f).
2) If the displacements of the X-ray source are small compared to the distance from the source to the plane of cut, then a differential length along the X-ray path (ds) can be replaced by the corresponding vertical differential length (dz_i).
3) Since the values of the linear attenuation coefficients, or at least its variations from point to point, are small, the exponential representing the attenuation of X-rays through matter can be approximated by the linear terms of its Taylor series expansion [7].

Considering these approximations, the final result is given in (1):

$$G(f_x,f_y) = I_b\,\delta(f_x,f_y) - \int_o^d H_i(f_x,f_y,z_i)\,F_\mu(f_x,f_y,z_i)\,dz_i \tag{1}$$

where $G(f_x,f_y)$ is the Fourier transform of the resulting image on the film, I_b is a constant, $\delta(f_x,f_y)$ is the Dirac delta function, $H_i(f_x,f_y,z_i)$ is the transfer function of the ith layer, at a distance z_i from the film, as given in (2), and $F_\mu(f_x,f_y,z_i)$ is the two-dimensional Fourier transform of the attenuation coefficients, $\mu(x_i,y_i,z_i)$, at depth z_i, as given in (3):

$$H_i(f_x,f_y,z_i) = \left[\frac{z_i-d}{z_i-D_2}\,\frac{D_1}{d}\right]^2 \iint I_o\left\{\frac{z_i-d}{z_i-D_2}\,\frac{D_1}{d}\,x,\ \frac{z_i-d}{z_i-D_2}\,\frac{D_1}{d}\,y\right\} e^{-j2\pi(f_x x+f_y y)}\,dx\,dy \tag{2}$$

$$F_\mu(f_x,f_y,z_i) = \iint \mu\left(\frac{d-z_i}{d}\,x,\ \frac{d-z_i}{d}\,y, z_i\right) e^{-j2\pi(f_x x+f_y y)}\,dx\,dy \tag{3}$$

The transfer function of the plane of cut is a constant and its impulse response is an impulse, as expected by intuition (cf. Figure 1). Equation (1) already suggests that the plane of cut can be changed by filtering the image. Indeed, suppose that there is interest in the plane at a depth $z_i=z_t$. Dividing both sides of (1) by $H(f_x,f_y) \triangleq H_t(f_x,f_y,z_t)$, the new overall transfer function for the layer at a depth z_t is a constant; thus this layer has become the new plane of cut. The overall transfer function of the plane previously in focus

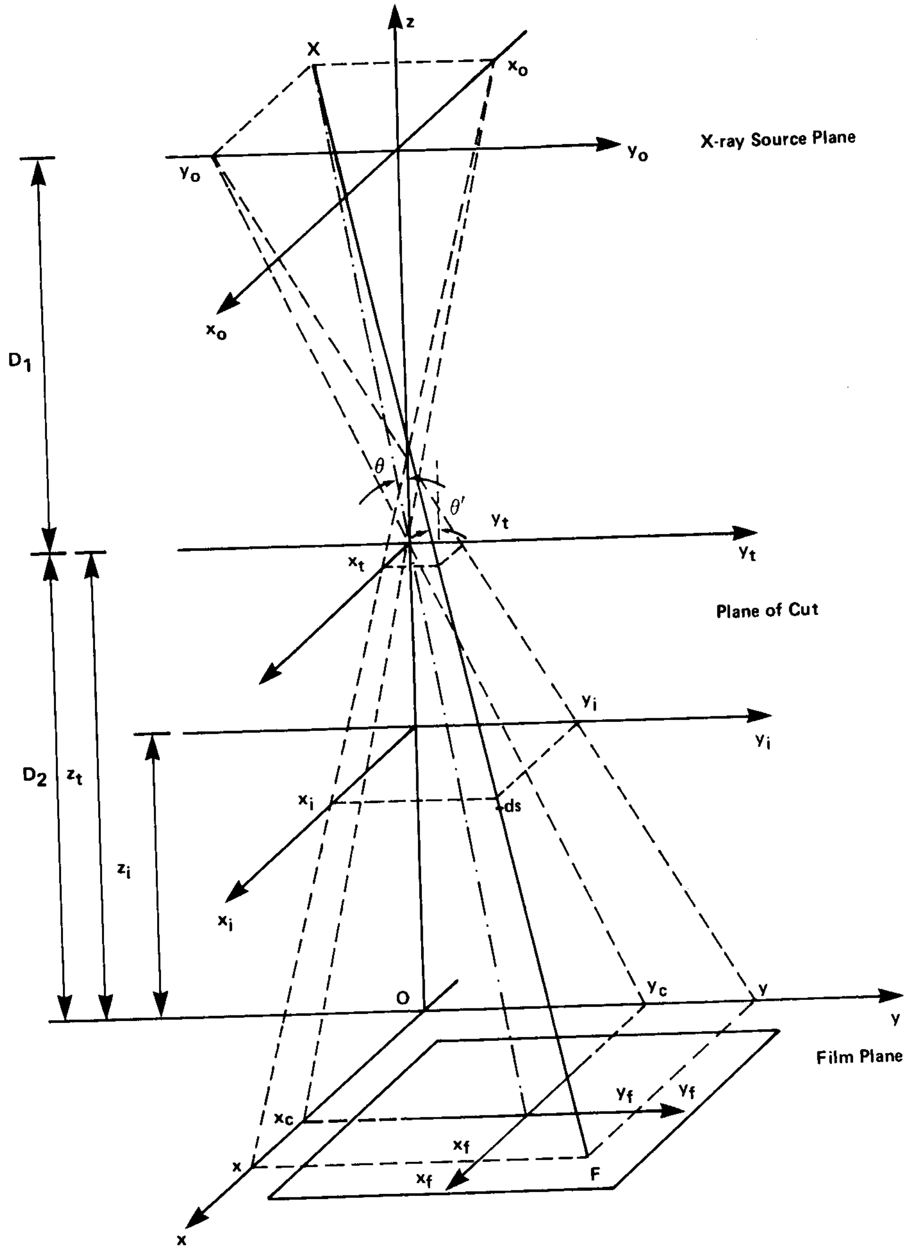

Figure 1. Diagram of standard tomography.

(z_t=D_2) is now $1/H(f_x,f_y)$. The overall transfer function for any other layer (i.e. at depth z_i) is $H_i(f_x,f_y,z_i)/H(f_x,f_y)$.

To derive the equation of conventional radiology, consider a radiologic system with focal spot intensity distribution $I_o(x_o,y_o)$ and film to focal-spot distance d. The diagram in Figure 1 still applies by letting D_2=0 and the movement of the point source of X-rays in standard tomography is replaced by the intensity distribution of the finite-size focal spot. Under these conditions all the derivations leading to (1) are still valid, but with D_2=0. It should be noted, however, that the exposure function $I_o(x_o,y_o)$ is substantially different, although mathematically it makes no difference. Thus, in conventional radiography the transfer function to be used in (1) is:

$$H_i(f_x,f_y,z_i) = \left[\frac{z_i-d}{z_i}\right]^2 \iint I_o\left\{\frac{z_i-d}{z_i}\,x,\ \frac{z_i-d}{z_i}\,y\right\} e^{-j2\pi(f_x x + f_y y)}\,dx\,dy \qquad (4)$$

Therefore, the mathematical models of standard tomography and conventional radiology are similar but with different transfer functions. In radiography none of the layer transfer functions is identically equal to a constant except in the limiting case that $z_i=0$ (film plane).

As before, the radiograph can be filtered so that the overall transfer function of one of the layers is equal to a constant, thus converting a radiograph into a tomogram. Hence, the equation of tomographic filtering for conventional radiography is the same as (1) but with the transfer function $H_i(f_x,f_y,z_i)$ given by (5).

$$H_i(f_x,f_y,z_i) = \frac{\left[\frac{z_i-d}{z_i}\right]^2 \iint I_o\left\{\frac{z_i-d}{z_i}\,x,\ \frac{z_i-d}{z_i}\,y\right\} e^{-j2\pi(f_x x+f_y y)}\,dx\,dy}{\left[\frac{z_t-d}{z_t}\right]^2 \iint I_o\left\{\frac{z_t-d}{z_t}\,x,\ \frac{z_t-d}{z_t}\,y\right\} e^{-j2\pi(f_x x+f_y y)}\,dx\,dy} \tag{5}$$

Consequently, it has been shown that by comparing the movement of a point X-ray source with a finite size focal spot and replacing the movement of the film in standard tomography by filtering a conventional radiograph, an analogy between standard tomography and tomographic filtering can be established.

The transfer function of the tomographic filter is, therefore, the inverse of the one shown in (4) with $z_i=z_t$, the depth of the desired plane of cut. Since the gain of an ideal inverse filter can become very high, it is necessary to design the filter carefully, as will be discussed later.

For evaluation purposes, more significant than the characteristics of the tomographic filter itself is the overall transfer function (5), including that of the original system. An analysis of it has shown that in the low-frequency region, the overall transfer function has low-pass characteristics for layers between the plane of cut and the focal spot [6]. On the other hand, between the plane of cut and the film the overall transfer function has high-pass characteristics [6]. This is illustrated in Figure 2, where for simplicity only one frequency axis is shown. These results could also be applied to the study of the effects of errors in the determination of the transfer function in inverse filtering problems, in general.

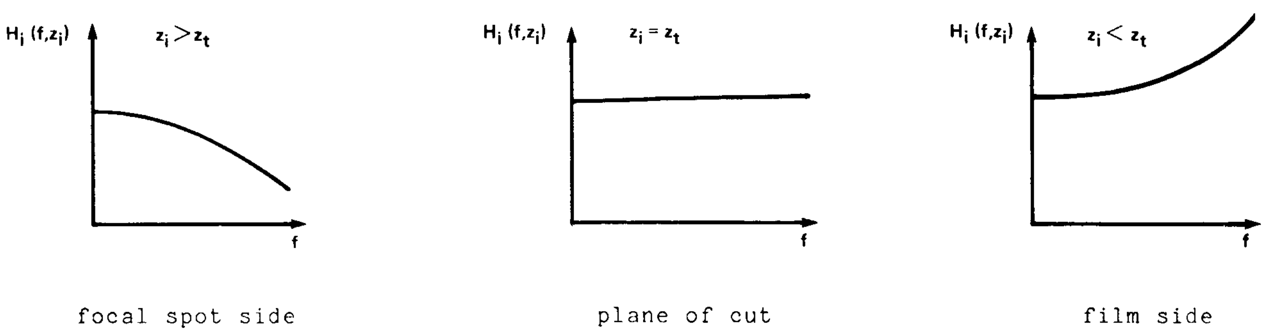

Figure 2. Characteristics of overall transfer function in a TFP.

Comparative evaluation of tomographic filters

A comparative assessment of tomographic filters was produced [6], taking as benchmarks two well established radiologic procedures: standard tomography and conventional radiology. This comparison took into account the different nature of the processes (otherwise same exposure function, same geometry, etc.) and, separately, the different dimensions normally associated with each one. The transfer functions in (2), (4) and (5) would contain all the information necessary to compare the various systems. However, they are inconvenient to calculate and compare. The first simplification is to ignore the phase transfer function and consider only the Modulation Transfer Function (MTF). Nevertheless, for ease of comparison single number parameters are preferred. The thickness of the tomographic layer, the rate of change of the MTF and the signal to noise ratios are considered here.

Thickness of the tomographic layer

In standard tomography the thickness of the cut is normally defined as the distance between two levels which have a tomographic blurring that is insufficiently large to be noticeable in the presence of the usual radiographic blurrings. This is a subjective definition and it depends on the relative amount of other blurrings such as those due to the focal-spot intensity distribution and patient movement. On the other hand, in a tomographic filtration process (TFP) the tomographic blur is based on the focal spot intensity distribution and the blur due to patient movement is negligible because the exposure time is very short.

Hence, the thickness of the cut depends on the extent of the movement of the X-ray source in tomography or the size of the focal spot in a TFP. It is more usual to give the exposure angle rather than the extension of the movement of the X-ray source (or size). The exposure angle is defined as the angle through which the projecting ray of a central point of the plane of cut 'moves' during the exposure. In tomography the exposure angle normally ranges from 1-5 degrees (in zonography) to 120-170 degrees (in transversal tomography) [8]. In conventional radiography, and therefore in a TFP, the exposure angle is determined by the size of the focal spot. With a typical focal spot size of 2 mm and focal spot to plane distance of 1000 mm, the exposure angle is about 0.1 degrees. Thus, in terms of the exposure angle a TFP would be closer to zonography than to any other tomographic technique.

When exposure angle is translated to thickness of cut, in standard tomography it is of the order of a few millimetres, in zonography it is of the order of a few centimetres and in a tomographic filtration process even larger. Due to the lack of experimental data conclusive results cannot be given for a TFP [6]. However, it is expected that by using visual workstations for interactive viewing (e.g. with zooming and magnification) the apparent thickness of cut in a TFP could become close to that of zonography. A TFP is an improvement over conventional radiography but it cannot achieve the thin cuts of standard tomography.

The rate of change of the Modulation Transfer Function (MTF)

A measure has been proposed to quantify the contrast between layers after they have been imaged on the film [6]. This is based on the rate of change of the transfer functions in (2), (4) and (5) from layer to layer for a specific type of exposure function I (x ,y). Quantitative results were obtained by assuming a Gaussian function. This may not be realistic, but it provides a good basis to compare the performance of the tomographic filtration process with that of standard tomography. When identical exposure functions are considered, the results showed that for layers between the focal spot and the plane at a distance $(d \cdot D_2)/(2 \cdot d - D_2)$ from the film, the transfer function in a TFP varies faster from layer to layer than in the equivalent system using standard tomography. It can be shown that this interval always contains the plane of cut $z_i = D_2$, hence in a region around the tomographic layer a TFP gives better contrast between layers than standard tomography. However, if the normal sizes of the exposure function are taken into consideration (i.e. about 500 mm in standard tomography and about 2 mm in TFP), the performance of standard tomography is by far better because the interval around the plane of cut is negligible.

The signal to noise ratios

The signal to noise ratio (SNR) is defined here as the ratio of the power of the signal from the tomographic layer if it was the only one present in the object and the power of the noise contributed by all other layers. This signal to noise ratio provides another measure of the contrast of the image of the plane of cut with respect to the others. The detailed derivation is given in [6], here only a summary is presented.

The objectbeing X-rayed, represented by the distribution of linear attenuation coefficients, is considered to be a random process. The power is given by the integral of its power spectral density function. To determine the signal to noise ratio the power component due to the image of the tomographic layer (P_t) and the noise power due to the other layers (P_n) are separated. It is also useful to separate the noise power due to layers between the anode and the tomographic layer (P_a) and the noise power due to layers between the tomographic layer and the film (P_f). Equation (6) shows how they are related.

$$P = P_t + P_n = P_t + P_a + P_f \qquad (6)$$

Formulae to calculate these powers can be found in [6]. The various signal to noise ratios can then be calculated as follows:

$$SNR = P_t/P_n \qquad (7)$$

$$SNR_a = P_t/P_a \tag{8}$$

$$SNR_f = P_t/P_f \tag{9}$$

$$SNR = \frac{SNR_a \bullet SNR_f}{SNR_a + SNR_f} \tag{10}$$

Equations (6) to (10) were calculated in about 4000 cases. Table 1 shows a representative sample of the results: the variation of the signal to noise ratio with respect to the nominal thickness of the tomographic layer. To calculate Table 1 the following parameters were assumed: d=1000 mm, z_t=500 mm, object of thickness 264 mm and positioned at equal distances from film and focal spot, object made of white noise bandlimited at 5 cycles/mm, and $I_o(x_o,y_o)=\exp(-100\cdot x_o^2 - 100\cdot y_o^2)$.

Table 1. The signal to noise ratios versus the thickness of the cut.

Thickness of the cut (mm)		4	20	40	100	200	240
Standard tomography	SNR	0.017	0.089	0.19	0.67	3.6	11.
	SNR_a	0.033	0.18	0.39	1.3	7.1	23.
	SNR_f	0.033	0.18	0.39	1.3	7.1	23.
Conventional radiography	SNR	0.015	0.081	0.18	0.60	3.1	9.8
	SNR_a	0.036	0.19	0.43	1.5	9.3	27.
	SNR_f	0.026	0.14	0.3	0.99	4.9	15.
Tomographic filtering	SNR	0.013	0.068	0.15	0.48	2.3	7.4
	SNR_a	0.045	0.24	0.55	2.1	13.	47.
	SNR_f	0.018	0.093	0.2	0.62	2.8	8.7

It is clear that the SNR (also SNR_a and SNR_f) increases with the thickness of the tomographic layer, as expected, because of its definition. Other results have shown that when the object is moved closer to the focal spot or the size of the exposure function increases, the SNR increases in standard tomography, but in conventional radiography and tomographic filtering it decreases. When the system parameters are the same (i.e. any column in Table 1) SNR_a is maximum for tomographic filtering and minimum for standard tomography. On the other hand SNR_f and SNR are maximum for standard tomography and minimum for tomographic filtering. This shows that tomographic filters perform better for layers in the object closer to the film. When the tomographic layer is closer to the focal spot the high pass effect on the layers on the side of the film produce the decrease in SNR through an increase in the noise power.

These measures give only an indication of the performance from a theoretical point of view. In practice, the object is very structured and the effects of noise due to other layers cannot be calculated statistically.

Implementation of tomographic filters

Tomographic filters can be implemented optically or digitally. The use of two-dimensional digital filters was preferred in this research because of their advantages, such as flexibility, accuracy and speed. Moreover, with digital radiography the realization of tomographic filtering is simplified.

From (5), the transfer function of the tomographic filter is the inverse of the transfer function for the layer of interest. Since the gain of an ideal inverse filter can become very high, it is necessary to provide some guards. Several filter structures have been compared in the literature [9]. In this work a simple technique was used which provides a means for hard-limiting the magnitude response of the inverse filter and cutting off the high frequencies dominated by noise [6], [10]. Two types of experiments were performed. Computer simulations using a computer simulation of the conventional radiology process, and experiments with actual radiographs of a fanthom chest.

Figure 3 shows the results of one simulation dealing with a two-dimensional object so that the effect of tomographic filtering at different depths could be observed.

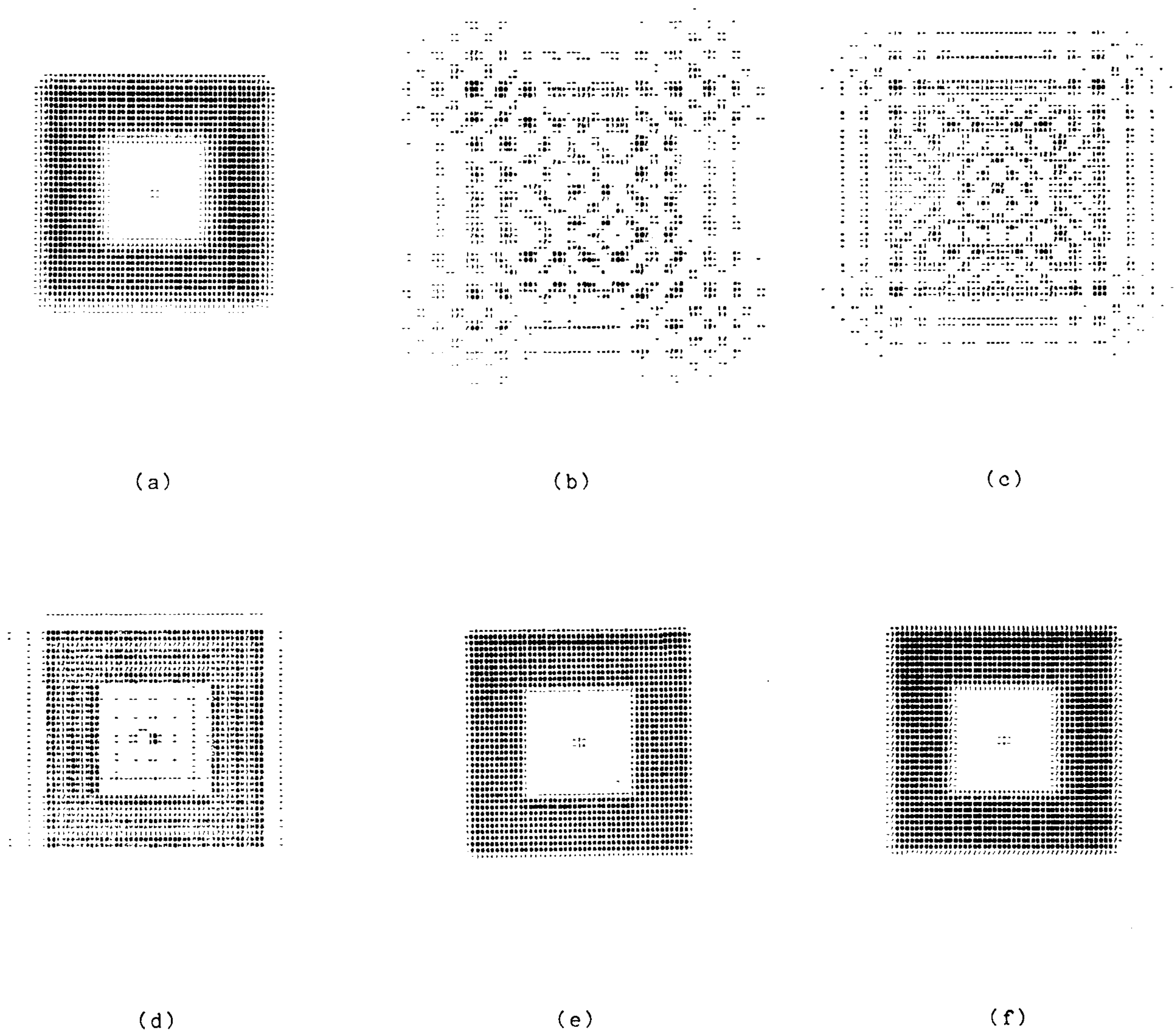

Figure 3. Examples of tomographic filtering results: (a) Simulated radiograph of a square-like annulus located 600 mm from the focal spot and 400 mm from the film. This image was processed with tomographic filters designed for layers at the following distances from the film: (b) 600 mm, (c) 550 mm, (d) 450 mm, (e) 400 mm, (f) 350 mm.

Figure 3 clearly shows the high pass effect when a filter designed for a layer is used on another layer located closer to the film. The breakdown of unwanted structures is dramatic, although in practical applications the added source of noise may hinder the view of other layers. The parameters of the tomographic filter (e.g. magnitude hard limit and cut-off frequency) can be chosen so that it has little effect on the image or the filter can be made more aggressive, until the image becomes dominated by noise. This phenomenon has some analogy to the inverse filtering problem if the inverse transfer function does not match the ideal, but it has some distinct peculiarities. It is complicated by the fact that there are many layers superimposed on the image, each affected by a different (overall) transfer function.

In practical applications the radiographic system can be characterized by a pin-hole image of the X-ray source and the geometry of the set-up. These are used to determine the transfer function of the desired plane of cut, as shown in the denominator of (5). More examples, details of the design and realization of tomographic filters and computer program listings can be found in [6].

Conclusions

A TFP is a technique for recovering three-dimensional information from a single radiograph. Theoretical comparisons of the performance of tomographic filters have been carried out. A TFP is an improvement over conventional radiography but it cannot achieve the thin cuts of standard tomography. The main advantage of a TFP is the lower radiation dose to the patient because only one radiograph is used, from which more images can be generated interactively. The main drawback of a TFP is the noise generated by the high-pass frequency response effect on the layers between the plane of cut and the film. It has been shown that the filter can be designed to have little effect on the radiograph, or if the technique is pushed too far, the image becomes dominated by noise. This effect has some analogy to the inverse filtering problem but has distinct peculiarities. With digital radiography the implementation of tomographic filters is greatly facilitated. It is recommended that clinical evaluations be developed to determine the usefulness of the technique from a medical point of view and find specific diagnostic applications.

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